Math 342, Winter 2009

REVIEW SHEET FOR MIDTERM 2

The exam focuses on the main concepts and topics of Chapter 6. There may be a few definitions on the exam. The most important definitions include:

orthogonal projection, orthogonal basis, least-squares solution, QR factorization, inner product.

A number of questions will require that you give reasons for your answers. These reasons will often involve a reference to a theorem. Theorems that have descriptive names attached to them are usually good candidates for a question. Examples:

Best Approximation Theorem, Orthogonal Decomposition Theorem

Definitions:

Length of a vector, orthogonal vectors, orthogonal set, orthogonal basis.
Orthogonal projection of $\mathbf{y}$ onto a line through $\mathbf{0}$
Orthogonal projection of $\mathbf{y}$ onto a subspace W, orthogonal complement of W.
QR factorization.
Least-squares solution, normal equations.
Design matrix, parameter vector, observation vector.

Theorems:

Theorem 2 (Pythagorean Theorem) – Know proof, Theorem 4.
Theorem 8 (The Orthogonal Decomposition Theorem).
Theorem 9 (The Best Approximation Theorem).
Theorem 12 (QR Factorization).
Theorem 13 (Normal Equations).
Theorem 15. Know basic calculation for proof of Theorem 14.

Important skills:

Compute length of vector, distance between vectors.
Determine if a set is orthogonal, normalize a vector.
Construct an orthonormal set from an orthogonal set. Know $||\mathbf{x}||^2 = \mathbf{x}^T\mathbf{x} = \mathbf{x} \cdot \mathbf{x}$.
Check a set for orthogonality.
Compute orthogonal projection onto a line (through $\mathbf{0}$) or a subspace.
Decompose a vector into a component in the direction of $\mathbf{u}$ and a component orthogonal to $\mathbf{u}$. Decompose a vector into the sum of a vector in a subspace W and a vector in $W^\perp$.
Find the vector in a subspace W that is closest to a specified vector.
Find the distance from a subspace to a specified vector.
Perform the Gram-Schmidt process on a linearly independent set of vectors.
Compute the QR factorization of a matrix with linearly independent columns.
Find a least-squares solution of $A\mathbf{x} = \mathbf{b}$. Compute the least-squares error.

Applications:

Construct a QR factorization of a matrix, use a QR factorization to produce a least-squares solution of $Ax = b$.
Find the least-squares line to fit a set of data, set up a design matrix for a least-squares fit to data by a specified equation, identify the parameter vector and the observation vector.
Write the normal equations for the linear model $\mathbf{y} = X\beta + \varepsilon$ (namely, $X^TX = X^T\mathbf{y}$).