Math 342, Self-Quiz 1

Mark each statement as True or False. Justify each answer.

a. If A is invertible and 1 is an eigenvalue for A, then 1 is also an eigenvalue for A^{-1}.

b. If A is row equivalent to the identity matrix I, then A is diagonalizable.

c. If A contains a row or column of zeros, then 0 is an eigenvalue of A.

d. Each eigenvalue of A is also an eigenvalue of A^2.

e. Each eigenvector of A is also an eigenvector of A^2.

f. Each eigenvector of an invertible matrix A is also an eigenvector of A^{-1}.

g. Eigenvalues must be nonzero scalars.

h. Eigenvectors must be nonzero vectors.

i. Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.

j. Similar matrices always have exactly the same eigenvalues.

k. Similar matrices always have exactly the same eigenvectors.

l. The sum of two eigenvectors of a matrix A is also an eigenvector of A.

m. The eigenvalues of an upper triangular matrix A are exactly the nonzero entries on the diagonal of A.

n. The matrices A and A^T have the same eigenvalues, counting multiplicities.

o. If a 5×5 matrix A has fewer than 5 distinct eigenvalues, then A is not diagonalizable.

p. There exists a 2×2 matrix that has no eigenvectors in $\mathbb{R}^2$.

q. If A is diagonalizable, then the columns of A are linearly independent.

r. A nonzero vector cannot correspond to two different eigenvalues of A.

s. A (square) matrix A is invertible if and only if there is a coordinate system in which the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is represented by a diagonal matrix.

t. If each vector $\mathbf{e}_j$ in the standard basis for $\mathbb{R}^n$ is an eigenvector of A, then A is a diagonal matrix.

u. If A is similar to a diagonalizable matrix B, then A is also diagonalizable.

v. If A and B are invertible $n \times n$ matrices, then AB is similar to BA.

w. An $n \times n$ matrix with n linearly independent eigenvectors is invertible.

x. If A is an $n \times n$ diagonalizable matrix, then each vector in $\mathbb{R}^n$ can be written as a linear combination of eigenvectors of A.