MATH 342, SELF-QUIZ 2

The following statements refer to vectors in $\mathbb{R}^n$ (or $\mathbb{R}^m$) with the standard inner product. Mark each statement True or False. Justify each answer.

a. The length of every vector is a positive number.

b. A vector $\mathbf{v}$ and its negative $-\mathbf{v}$ have equal lengths.

c. The distance between $\mathbf{u}$ and $\mathbf{v}$ is $\|\mathbf{u} - \mathbf{v}\|$.

d. If r is any scalar, then $\|r\mathbf{v}\| = r\|\mathbf{v}\|$.

e. If two vectors are orthogonal, they are linearly independent.

f. If $\mathbf{x}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$, then $\mathbf{x}$ must be orthogonal to $\mathbf{u} - \mathbf{v}$.

g. If $\|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

h. If $\|\mathbf{u} - \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

i. The orthogonal projection of $\mathbf{y}$ onto $\mathbf{u}$ is a scalar multiple of $\mathbf{y}$.

j. If a vector $\mathbf{y}$ coincides with its orthogonal projection onto a subspace W, then $\mathbf{y}$ is in W.

k. The set of all vectors in $\mathbb{R}^n$ orthogonal to one fixed vector is a subspace of $\mathbb{R}^n$.

l. If W is a subspace of $\mathbb{R}^n$, then W and $W^\perp$ have no vectors in common.

m. If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal set and if c_1, c_2, c_3 are scalars, then $\{c_1\mathbf{v}_1, c_2\mathbf{v}_2, c_3\mathbf{v}_3\}$ is an orthogonal set.

n. If a matrix U has orthonormal columns, then $UU^T = I$.

o. A square matrix with orthogonal columns is an orthogonal matrix.

p. If a square matrix has orthonormal columns, then it also has orthonormal rows.

q. If W is a subspace, then $\|\operatorname{proj}_W \mathbf{v}\|^2 + \|\mathbf{v} - \operatorname{proj}_W \mathbf{v}\|^2 = \|\mathbf{v}\|^2$.

r. A least-squares solution of $A\mathbf{x} = \mathbf{b}$ is the vector $A\hat{\mathbf{x}}$ in $\operatorname{Col} A$ closest to $\mathbf{b}$, so that $\|\mathbf{b} - A\hat{\mathbf{x}}\| \leq \|\mathbf{b} - A\mathbf{x}\|$ for all $\mathbf{x}$.

s. The normal equations for a least-squares solution of $A\mathbf{x} = \mathbf{b}$ are given by $\hat{\mathbf{x}} = (A^TA)^{-1}A^T\mathbf{b}$.