Theory Appendix for:

"Buyer-Seller Relationships in International Trade: Evidence from U.S. State Exports and Business-Class Travel"

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Abstract

This appendix derives the relation between the aggregate fixed export costs (i.e., costs paid by all firms exporting to a market) and the volume and composition of bilateral exports in a standard multi-sector heterogeneous firms model of trade similar to Chaney (2008). The purpose is to show that allowing for cross-sector differences in the fixed cost of exporting does not change the paper's prediction of a positive relation between the aggregate fixed export costs and the trade-share weighted average parameter A (described by equation (18) in the text).

1 Set-up and Assumptions

Throughout this appendix I will use the same notation as in the main text of the paper. The main elements of the model set-up can be briefly summarized as follows:

- *Utility*: Cobb-Douglas across H differentiated sectors, CES across varieties within a sector
- *Firm heterogeneity*: firms differ in productivity φ, which is Pareto distributed with cdf $G(\varphi; \kappa)$
- *Firm costs*: the marginal cost adjusted for 'standard' quality is $1/\varphi$ and the fixed cost of exporting to destination j is F_{sjh}

The key assumptions that transforms the model described in the paper into a standard multi-sector heterogeneous firms model of trade (in the spirit of Chaney (2008)) are given by equation (17) in the main text, and reproduce here:

Assumption (A1): $\theta_h = 0, \ \forall h$
Assumption (A2): $F_{sjh} = c_{sj}\bar{i}_{jh}$

Assumption (A1) states that the relationship intensity parameter is zero across sectors, which implies that there is no firm level choice regarding in-person business meetings between trading partners. It is easy to verify in the main text that under assumption (A1) products loose their relationship-specific attribute, i.e. $\lambda_{sjh} = 1$, and as a result exporters have no incentive to interact with foreign partners (i.e., $i^*_{sjh} = 0$). This eliminates the endogenous component of the two-part fixed export cost, i.e., $c_{sj} i^*_{sjh} = 0$.

Assumption (A2) essentially gives a particular interpretation to the fixed cost of exporting: in this version of the model, F_{sjh} is viewed as an exogenous level of business travel imposed on any firm in sector h that intends to export to foreign market j (with c_{sj} representing the per unit travel cost).

I proceed next by describing the partial equilibrium of this 'exogenous travel requirement' model. Given that this alternative version of the model is almost identical to Chaney(2008), I will be brief the mathematical derivations and explanations. The focus will be on exports from a given U.S. region s to a destination country j in a sector h, keeping the third country exporters (J in number) in the background. The analysis is a partial equilibrium in that total income of country j (Y_j), the CES price index of sector h in country j (P_{jh}), and the wages in the exporting state s (w_s), are not explicitly derived but taken as given. The only equilibrium variables derived in this appendix are the product price, the firm export revenue, the CES price index of across U.S. exports in sector h in country j, the sector level bilateral exports, and the number of active exporters within each sector.

2 Firm's Problem

Demand for a variety. Utility maximization by consumers in country j delivers the demand:

$$x_{sjh}(\varphi) = \big(\tau_{sj} p_{sh}(\varphi)\big)^{-\sigma} \frac{\mu_{jh} Y_j}{P^{\text{US}}_{jh}} \gamma_{jh} \tag{1}$$

with

$$P^{\text{US}}_{jh} = \sum_{i=1}^{S} M_i \int_{\overline{\varphi}_{ijh}} \big(\tau_{sj} p_{sh}(\varphi)\big)^{1-\sigma} dG(\varphi) \quad \text{and} \quad \gamma_{jh} \equiv \frac{P^{\text{US}}_{jh}}{P_{jh}} \tag{2}$$

P_{jh} represents the CES price index for sector h and country j, computed over all varieties sold in destination market j. For reasons that will become clear later on, I have multiplies and divided the demand equation by the CES price index computed only across U.S. exports to country j in sector h (i.e., P^{US}_{jh}).

Price. Each firm chooses the product price, adjusted for 'standard' quality, as a constant mark-up over the marginal cost:

$$p_{sjh}(\varphi) = \frac{\sigma}{\sigma - 1} \varphi^{-1} w_s \tag{3}$$

Firm export revenues and profits. Given demand and the product price, the export revenue of a firm with productivity φ is:

$$r_{sjh}(\varphi) = \left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} \left(\frac{\tau_{sj} w_s}{\varphi}\right)^{1-\sigma} \frac{\widetilde{\mu}_{jh} Y_j}{P^{\text{US}}_{jh}} \tag{4}$$

with $\widetilde{\mu}_{jh} \equiv \mu_{jh}\gamma_{jh}$.

Then, firm export profits are:

$$\pi_{sjh}(\varphi) = \frac{r_{sjh}(\varphi)}{\sigma} - F_{sjh} \tag{5}$$

3 Partial Equilibrium

Zero profit condition and productivity cutoff.

$$\pi_{sjh}(\overline{\varphi}) = 0 \qquad \Rightarrow \qquad \overline{\varphi}_{sjh} = \alpha_1(\tau_{sj}w_s)\left(\frac{1}{F_{sjh}}\frac{\widetilde{\mu}_{jh}Y_j}{P^{\text{US}}_{jh}}\right)^{1/(1-\sigma)} \tag{6}$$

with $\alpha_1 = \left(\sigma^{1/(\sigma-1)}\frac{\sigma}{\sigma-1}\right)$ a constant.

US-only CES Price Index. Given that in equilibrium only firms above the productivity threshold $\overline{\varphi}$ export, we can re-calculate the CES price index from equation (2) conditional on the income level Y_j in the foreign market and wages w_i in the U.S. source region as:

$$\begin{aligned}
P^{\text{US}}_{jh} &= \sum_{i=1}^{S} M_i \int_{\overline{\varphi}_{ijh}} \left(\frac{\sigma}{\sigma-1}\varphi^{-1}w_i\right)^{1-\sigma} dG(\varphi) \\
&= \sum_{i=1}^{S} M_i \left(\frac{\sigma}{\sigma-1}\tau_{ij}w_i\right)^{1-\sigma} \frac{(\overline{\varphi}_{ijh})^{\sigma-\kappa-1}}{\kappa-\sigma+1} \\
&= \delta_1\left(\widetilde{\mu}_{jh}Y_j\right)^{1+\frac{1-\sigma}{\kappa}} \Phi_{jh}^{1-\sigma}
\end{aligned} \tag{7}$$

with $\Phi_{jh} = \left(\sum_{i=1}^{S} M_i\left(\tau_{ij}w_i\right)^{-\kappa} F_{ijh}^{-\frac{\kappa-(\sigma-1)}{\sigma-1}}\right)^{-1/\kappa}$ capturing the 'multilateral resistance' term and $\delta_1 = \left[(\sigma)^{1-\frac{\kappa}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)^{-\kappa}\left(\frac{1}{\kappa-\sigma+1}\right)\right]^{\frac{\sigma-1}{\kappa}}$ a constant.

Number of active firms. The equilibrium number of exporters from state s to destination country j is given by: $N_{sjh} = M_s\left(1 - G(\overline{\varphi}_{ijh}\right)$. Using the Pareto distribution assumption and substituting for the productivity threshold using equation (6), one arrives at the following solution:

$$N_{sjh} = \delta_2 M_s(\tau_{sj}w_s)^{-\kappa}\left(\frac{\widetilde{\mu}_{jh}Y_j}{\Phi_{jh}}\right)^{\kappa/\sigma-1}(F_{sjh})^{-\frac{\kappa}{\sigma-1}} \tag{8}$$

with $\delta_2 = \frac{\kappa-\sigma+1}{\kappa\sigma}$ a constant.

Aggregate Exports at Sector Level. They are computed by summing export revenues of all the active exporters:

$$X_{sjh} = M_s \int_{\overline{\varphi}_{ijh}} r_{sjh}(\varphi) dG(\varphi)$$

Substituting for firm revenues r_{sjt} using equation (4), and for the productivity threshold using equation (6), one gets the following expression:

$$X_{sjh} = \alpha_2 M_s \left(\frac{\widetilde{\mu}_{jh} Y_j}{P^{\mathrm{US}}_{jh}} \right)^{\frac{\kappa}{\sigma-1}} (\tau_{sj} w_s)^{-\kappa} (F_{sjh})^{1-\frac{\kappa}{\sigma-1}}$$

with $\alpha_2 = \left[(\sigma)^{1-\frac{\kappa}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-\kappa} \left(\frac{1}{\kappa-\sigma+1}\right) \right]$ a constant.

Further, substituting for the (US-only) CES price index using equation (7), the sector level aggregate exports become:

$$X_{sjh} = M_s(\widetilde{\mu}_{jh} Y_j) \left(\tau_{sj} w_s / \Phi_{jh} \right)^{-\kappa} F_{sjh}^{1-\frac{\kappa}{\sigma-1}} \tag{9}$$

Dividing aggregate exports by the number of active exporters one gets the following identity (standard in heterogeneous firms models with Pareto distributed productivities):

$$F_{sjh} = A \frac{X_{sjh}}{N_{sjh}}, \qquad A = \left(\frac{\kappa - \sigma + 1}{\kappa\sigma}\right) < 1/\sigma \tag{10}$$

Aggregate Level of Business Meetings. Defining the sector level business meetings as the total travel requirement F_{sjh} incurred by all exporters in equilibrium, then using assumption (A2) we can re-write the above expression to get: $I_{sjh} \equiv N_{sjh} \bar{i}_{jh} = A X_{sjh} / c_{sj}$. Further, summing the sector level business meetings across all the H differentiated sectors, we get an expression for the aggregate volume of business meetings incurred between state s and import country j:

$$I_{sj} \equiv \sum_h N_{sjh} \bar{i}_{jh} = \overline{A}^w \frac{X_{sj}}{c_{sj}}, \qquad \overline{A}^w \equiv \left(\sum_h A \frac{X_{sjh}}{X_{sj}} \right) \tag{11}$$

Equation (11) provides a direct relation between aggregate fixed export costs and the trade share weighted average parameter A, with A summarizing the production and market structure across industries (conditional on total bilateral exports).

Since both export shares (X_{sjh}/X_{sj}) and the number of firms (N_{sjh}) are endogenous variables that vary across sectors, the main challenge in predicting the relation between the (trade-share weighted) industry structure parameter $\overline{A}^w$ and the bilateral business travel flows is to understand how the two variables covary, particularly as a result of cross-sector differences in fixed costs.

4 Comparative Statics: Evaluating changes in $\overline{A}^w$ and I_{sj} from variation in fixed costs

In the comparative statics exercises of this section, I assume that each state is small relative to the world, and therefore disregard (unless otherwise mentioned) the indirect effects of a change in fixed costs working via the overall CES price index.

The analysis distinguishes two cases based on the assumption about parameter A, that is, whether it is considered identical or different across sectors.

Case 1: $A_h = A, \sigma_h = \sigma, \kappa_h = \kappa \quad \forall h$**.**

This case is maintained throughout the main text of the paper. From equation (11), it is straightforward to see that with identical production and market structures, the following holds: $\overline{A}^w = A$. This further implies:

$$I_{sj} = A\frac{X_{sj}}{c_{sj}} \qquad \Rightarrow \qquad dlnA = 0 \ ; \ dlnI_{sj} = dlnX_{sj} - dlnc_{sj} \tag{12}$$

When the industry structure parameter A is identical across sectors, conditional on aggregate bilateral exports, the level of A directly determines the level of bilateral fixed costs (i.e., $\sum_h N_{sjh}F_{sjh} = \sum_h N_{sjh}\bar{i}_{jh} \equiv I_{sj}$, conditional on c_{sj}). In addition, the changes in sector level exports sum up exactly to the aggregate change in total business meetings, holding constant the unit travel costs.

Notice that this outcome does not depend on any assumption regarding the level of the exogenous travel requirement across sectors $\bar{i}_{jh}$. Differences in sector level fixed export costs necessarily imply that business meetings will vary across sectors:[1]

$$\frac{\partial lnI_{sjh}}{\partial lnF_{sjh}} = \frac{\partial lnN_{sjh}}{\partial lnF_{sjh}} + 1 = \frac{\partial lnX_{sjh}}{\partial lnF_{sjh}} = -\frac{\kappa}{\sigma - 1} + 1 < 0 \tag{13}$$

In fact, sectors facing higher fixed export costs are responsible for relatively fewer business meetings (a result that follows from the fact that the elasticity of the number of firm with respect to the fixed export cost is greater than one in absolute value). However, what is important to note is that independent of the variation in export *levels* and in the number of exporters across sectors, the *change* in export volumes exactly reflects the change in aggregate fixed costs within each sector. This means that under the assumption of equal A parameters across sectors, the aggregate level of export is a sufficient statistic in a regression model to account for differences in fixed export costs across sectors.

Case 2: $A_h \neq A_g, \sigma_h \neq \sigma_g, \kappa_h \neq \kappa_g \quad \forall h \neq g$

In this case, equation (11) becomes:

[1] I have used equations (8) and (9) for doing the calculations, and the finite average sales condition (i.e., $\kappa > \sigma - 1$) to sign the derivatives.

$$I_{sj} = \overline{A}^w \frac{X_{sj}}{c_{sj}}, \qquad \text{with } \overline{A}^w = H \cdot cov\left(A_h, \frac{X_{sjh}}{X_{sj}}\right) + \frac{1}{H}\sum_h A$$

$$\Rightarrow \quad dlnI_{sj} = dlnX_{sj} + dln\overline{A}^w - dlnc_{sj} \tag{14}$$

When industry parameters vary across sectors, the key statistic needed to fully explain the impact of industry structure on aggregate business meetings is the covariance between A and the export share of that sector. If a larger fraction of bilateral exports falls in high A industries (i.e., sectors with low elasticity of substitution σ, and/or low variance in productivities across firms within industry – high κ), then the (trade-share weighted) average value of A will have a positive effect on aggregate bilateral business meetings. On the other hand, if a larger fraction of bilateral exports falls in low A sectors, then the impact of the average industry structure on aggregate business meetings is ambiguous (as it depends on the value of the covariance relative to the simple average of A across sectors).

In deriving a formal relation showing how differences in sector level fixed export costs map into changes in $\overline{A}^w$ (because of changes in export shares), and then further impact the aggregate fixed cost of trade (I_{sj}), the following assumptions simplify the analysis greatly:

i). firm productivity is Pareto distributed
ii). the fixed cost of exporting (for firms across U.S. regions) is separable into a bilateral (s-j) and industry-importer (h-j) specific components.[2] This consition is satisfies by assumption (A2).

Under these general assumptions it can be shown that the industry-importer specific component of the fixed export cost, $\bar{i}_{jh}$, impacts the sector level exports X_{sjh} both directly, via the fixed export cost F_{sjh}, and indirectly, via the CES price index P^{US}_{jh}, in exactly the same proportion. To see this, substituting assumption (A2) into the (US-only) CES price index in equation (7), and factoring out the industry-importer component we get:

$$P^{\text{US}}_{jh} = \delta_1 \left(\frac{\widetilde{\mu}_{jh} Y_j}{\bar{i}_{jh}}\right)^{1+\frac{1-\sigma}{\kappa}} \widetilde{\Phi}^{1-\sigma}_{jh} \tag{15}$$

with $\widetilde{\Phi}_{jh} = \left(\sum_{i=1}^{S} M_i \big(\tau_{ij} w_i\big)^{-\kappa} c_{ij}^{-\frac{\kappa-(\sigma-1)}{\sigma-1}} \right)^{-1/\kappa}$ and δ_1 a constant (given by equation (7)).

Similarly, substituting assumption (A2) into equation (9) and using the fact that $\Phi_{jh} = \widetilde{\Phi}_{jh}\big(\bar{i}_{jh}\big)^{\frac{1}{\sigma-1}-\frac{1}{\kappa}}$, the expression for sector level exports becomes:

$$X_{sjh} = M_s(\widetilde{\mu}_{jh} Y_j)\big(\tau_{sj} w_s / \widetilde{\Phi}_{jh}\big)^{-\kappa} c_{sj}^{1-\frac{\kappa}{\sigma-1}} \tag{16}$$

and relative exports between two sectors h and g is:

[2] It is important to mention that for deriving the predictions in this appendix, this separability assumption need not apply to all firms in all economies but just to U.S. firms. This is because in the analysis, I have separated the impact of the fixed export cost on the U.S.-only CES price index from the impact of third country fixed export cost on the overall CES price index in import market j.

$$\frac{X_{sjh}}{X_{sjg}} = \frac{\widetilde{\mu}_{jh}}{\widetilde{\mu}_{jg}} \left(\frac{\widetilde{\Phi}_{jh}}{\widetilde{\Phi}_{jh}} \right)^{\kappa_h - \kappa_g} \left(c_{sj} \right)^{-\left(\frac{\kappa_g}{\sigma_g - 1} - \frac{\kappa_g}{\sigma_g - 1} \right)} \tag{17}$$

Notice that now relative exports depend only on the bilateral component of the fixed export cost, c_{sj}; that is, the relative size of the fixed cost of exporting across sectors, F_{sjh}/F_{sjg}, *does not* affect relative exports through industry-specific effects. So now, we can use the results in Chaney (2008) on the elasticity of exports with respect to trade costs to understand how a change in the level of the fixed export costs, c_{sj}, impacts the trade-share weighted average parameter $\overline{A}^w$ and also the aggregate bilateral business travel flows I_{sjh}. The key feature that makes the analysis tractable without any further assumptions is the fact that now a change in fixed costs can occur only through changes in travel costs c_{sj}, which affects uniformly *all* sectors h.

Taking derivatives of relative exports in equation (17) with respect to relative fixed export costs F_{sjh}/F_{sjg} we get:

$$\begin{aligned} \frac{\partial ln\left(X_{sjh}/X_{sjg}\right)}{\partial ln\left(F_{sjh}/F_{sjg}\right)} = \frac{\partial ln\left(X_{sjh}/X_{sjg}\right)}{\partial ln c_{sj}} &= -\frac{\kappa_h}{\sigma_h - 1} + \frac{\kappa_g}{\sigma_g - 1} \\ &= -\frac{1}{1 - \sigma_h A_h} + \frac{1}{1 - \sigma_g A_g} \\ &= -\frac{\sigma_h A_h - \sigma_g A_g}{(1 - \sigma_h A_h)(1 - \sigma_g A_g)} \end{aligned} \tag{18}$$

where $\sigma_h A_h < 1, \ \forall h$ holds because $k_h > \sigma_h - 1$ by assumption. This shows that the relative change in sector level exports coming from a fixed cost shock depends on the relative size of the industry structure parameters.

What is the implication for the change in $\overline{A}^w$?
For expositional simplicity, assume the economy is made of two sectors h and g, so that:

$$\begin{aligned} \overline{A}^w \equiv A_h \frac{X_{sjh}}{X_{sj}} + A_g \frac{X_{sjg}}{X_{sj}} &= A_h + (A_g - A_h) \frac{X_{sjg}}{X_{sj}} \\ &= A_h + (A_g - A_h) \left(1 + \frac{X_{sjh}}{X_{sjg}} \right)^{-1} \end{aligned} \tag{19}$$

Then:

$$\frac{\partial \overline{A}^w}{\partial (X_{sjh}/X_{sjg})} = (A_h - A_g) \left(1 + \frac{X_{sjg}}{X_{sjh}} \right)^2 \tag{20}$$

Combining equation (18) and (20), it becomes clear that a change in the bilateral fixed export costs leads to a change in $\overline{A}^w$ in the opposite direction.[3] However, recall from equation (13) that sector level business meetings also respond negatively to a change in fixed cost. Putting the two pieces

[3]This implicitly relies on the assumption that σ_h, σ_g take values such that $\frac{A_h}{A_g} > \frac{\sigma_g}{\sigma_h}$.

of information together, I conclude that under assumption (A2), differences in fixed export costs F_{sjh} and in market and production structure across sectors and destination markets (i.e., A, κ, σ) do not revert the prediction that the (trade-share weighted average) market structure parameter $\overline{A}^w$ and aggregate business travel I_{sjh} covary positively. Furthermore, given a negative relation between $\overline{A}^w$ and HHI_{sjh}, this implies a negative correlation between HHI_{sjh} and I_{sjh}.

General discussion. In the absence of the separability assumption regarding the fixed costs of exporting given by (A2), it is very challenging and maybe impossible to reach an unambiguous conclusion regarding the covariance between $\overline{A}^w$ and I_{sjh}. To illustrate this point, consider as an example the following scenario: fixed cost F_{sjh} are initially identical across all sectors but increase in one particular sector h. Since $\frac{\partial lnI_{sjh}}{\partial lnF_{sjh}} = \frac{\partial lnX_{sjh}}{\partial lnF_{sjh}} < 0$, the share of sector h in bilateral exports X_{sj} falls (as the export volumes in all other sectors do not change), but so do business meetings in that sector I_{sjh}. However:

- if sector h has a high A value (i.e., *positive correlation* between $\mathrm{F_h}$ and $\mathrm{A_h}$), then the (trade-share weighted) average A falls, which implies a *positive* coefficient of $\overline{A}^w$ on aggregate business meetings I_{sj}.
- if sector h has a low A value (i.e., *negative correlation* between $\mathrm{F_h}$ and $\mathrm{A_h}$), then the (trade-share weighted) average A increases, which implies a *negative* coefficient of weighted average A on aggregate bilateral business meetings I_{sj}.

This example reveals that when industry parameters vary across sectors (i.e., $\mathrm{A_h}$'s are different), in order to understand the impact of the average industry structure parameter $\overline{A}^w$ on aggregate bilateral business meetings one would need to know the covariance between $\mathrm{A_h}$ and the industry-specific component of the fixed costs $\mathrm{F_h}$ across sectors.