Math 307
Homework Due Wednesday, April 15

In the following, "Problem Set" refers to the set in the xerox copies I handed out in class.

1. Problem Set 8: #1, 10.
2. Problem Set 9: #7, 10, 12, 13, 16, 17, 18.
3. Problem Set 10: #3, 4, 8, 10.
4. Solve the equation $(7 \times_{11} x) +_{11} 10 = 4$ in $\mathbb{Z}_{11}$ (and, as always in this course, explain your methods).
5. Find all solutions to the equation $x^2 + 4x + 10 = 0$ in $\mathbb{Z}_{11}$. (Hint: First, complete the square. Next, make a table of all squares in $\mathbb{Z}_{11}$. How does this help?)
6. For what primes p is the element $p - 1$ a perfect square in $\mathbb{Z}_p$? Investigate this question by working out the cases $p = 2$, $p = 3$, $p = 5$, $p = 7$, $p = 11$, $p = 13$, $p = 17$, and $p = 19$. See if you notice any patterns, and try to make a conjecture.
7. Consider a sum of three consecutive squares (like $7^2 + 8^2 + 9^2$). What do you get when you reduce this mod 3 (that is, when you compute the remainder when divided by 3)? Pick another sum of three consecutive squares and try it again. Try it one more time. State a conjecture, and see if you can prove it.
8. What happens when you take a sum of *four* consecutive squares and reduce it mod 4? Again, do some experimentation, make a conjecture, and see if you can prove it.