
Math 307
Homework Due Wednesday, April 22

In the following, “Problem Set” refers to the set in the xerox copies I handed out in class. As with all problems in this class, you should give explanations of your answers—so in #5 and #6 below, just stating the numerical answers is not sufficient.

1. Problem Set 12: #4, 6, 7, 12, 16.
2. Problem Set 13: #6, 8, 9, 14, 16.
3. Problem Set 14: #1. Also, determine the truth value of each statement in #4; if a “for all” statement is false, give an example showing it to be false, and if a “for some” statement is true, give an example showing it to be true.
4. In the textbook, do Exercises 2.4, 2.9, 2.11, 2.31, 2.48 with the following additions:
 - In 2.4 and 2.9, write both the statement and its negation using quantifiers and logical operators. Write the negation as a statement that doesn’t have a \sim in front of it.
 - In 2.11, the intention is that the “dollar” is a dollar *coin*.
 - In 2.31, also write each statement using quantifiers and logical operators.
5. Determine the remainder when 8^{1000} is divided by 7. Do the same for 9^{1000} and 10^{1000} .
6. Let $N = (6 \cdot 10^{100}) + (5 \cdot 10^{99}) + (7 \cdot 10^{98}) + (4 \cdot 10^2) + (3 \cdot 10) + 1$. What is the remainder when N is divided by 11?
7. Take a circle and draw n points on it, then draw line segments connecting every pair of the chosen points. We say the chosen points are “generic” if there are no points inside the circle where three or more lines intersect.
Let R_n be the number of regions that the circle is divided into by the line segments corresponding to n generic points. Determine the value of R_n for all $n \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. (To check that you are doing things right, you should get $R_5 = 16$.)
Warning: For the larger numbers in this set it will be hard to determine R_n by simple counting. Try to understand the difference $R_{n+1} - R_n$. How many new regions are created when you add in one more point?
8. I’m thinking of a positive integer with the following properties:
 - When it is divided by 5, the remainder is 2.
 - When it is divided by 7, the remainder is 3.
 - When it is divided by 11, the remainder is 1.

What is the smallest number I could be thinking of?