Math 307

Solutions to selected problems in HW5

1. #4a) This statement is true.

   Proof:

   (1) Let $x \in M_2 \cap M_3$.
   (2) Then $2|x$ and $3|x$.
   (3) So there is a $k \in \mathbb{N}$ such that $x = 2k$.
   (4) But then $3|2k$, so by the "fact" given at the beginning of the problem we know $3|2$ or $3|k$.
   (5) Since 3 does not divide 2, we can conclude $3|k$.
   (6) So $k = 3l$ for some $l \in \mathbb{N}$.
   (7) Then $x = 2k = 2(3l) = 6l$, hence $6|x$.
   (8) So $x \in M_6$.
   (9) Hence $M_2 \cap M_3 \subseteq M_6$.
   (10) Now let $z \in M_6$.
   (11) Then $6|z$, so $z = 6k$ for some $k \in \mathbb{N}$.
   (12) Then $z = 2 \cdot (3k)$, so $2|z$, so $z \in M_2$.
   (13) Similarly, $z = 3 \cdot (2k)$, so $3|z$, so $z \in M_3$.
   (14) Hence $z \in M_2 \cap M_3$.
   (15) So $M_2 \cap M_3 \subseteq M_6$.
   (16) Therefore $M_2 \cap M_3 = M_6$.

2. #5 bi) This is false. Let $f\colon \{1,2\} \to \{P\}$ be the function given by $f(1) = P$ and $f(2) = P$. Let $A = \{1\}$ and $B = \{2\}$. Then $A \cap B = \emptyset$, so $f(A \cap B) = f(\emptyset) = \emptyset$. But $f(A) = \{P\}$ and $f(B) = \{P\}$, so $f(A) \cap f(B) = \{P\}$. Observe that $f(A \cap B) \neq f(A) \cap f(B)$.

3. #6 biii) This is false. Let $f\colon \{1,2\} \to \{3,4\}$ be given by $f(1) = 3$ and $f(2) = 3$. Let $A = \{3,4\}$. Then $I_f(A) = \{1,2\}$, and $f(I_f(A)) = f(\{1,2\}) = \{3\}$. Observe that $f(I_f(A)) \neq A$.

4. #6 biv) This is false. Let $f\colon \{1,2\} \to \{3\}$ be given by $f(1) = 3$ and $f(2) = 3$. Let $C = \{1\}$. Then $f(C) = \{3\}$, and $I_f(f(C)) = I_f(\{3\}) = \{1,2\}$. Notice that $f(I_f(C)) \neq C$.

5. The statements in #5bii and #6bi,ii are all true.