Errata for *Markov Chains and Mixing Times*

Page 6, first display [A. Holroyd, 1/9/2009]. This should be

$$\mu(A) = \sum_{x \in A} \mu(x).$$

Page 12, Proposition 1.14 [F. Nazarov, 3/19/09]. Part (ii) requires the uniqueness of the stationary distribution, which is proven in Section 1.5.4, immediately following Proposition 1.14. Therefore, it should be removed from Proposition 1.14, and the following should be added after Corollary 1.17:

Proposition. *If π is the unique solution to $\pi = \pi P$ for an irreducible transition matrix P, then $\pi(x) = 1/\mathbf{E}_x \tau_x^+$.*

PROOF. Let $\tilde{\pi}_z(y)$ equal $\tilde{\pi}(y)$ as defined in (1.19), and write $\pi_z(y) = \tilde{\pi}_z(y)/\mathbf{E}_z \tau_z^+$. Proposition 1.14 implies that π_z is a stationary distribution, and Corollary 1.17 implies π_z does not depend on z. Thus if $\pi(y) := \pi_z(y)$ for some (and hence all z), then

$$\pi(x) = \pi_x(x) = \frac{\tilde{\pi}_x(x)}{\mathbf{E}_x \tau_x^+} = \frac{1}{\mathbf{E}_x \tau_x^+}\,.$$

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Page 13, Equation (1.27) [F. Nazarov, 3/19/09] . This should be

$$\mathbf{P}_{x_0}\{(X_{\tau+1}, X_{\tau+2}, \ldots, X_{\tau+\ell}) \in A \mid \tau = k \text{ and } (X_1, \ldots, X_k) = (x_1, \ldots, x_k)\} \\ = \mathbf{P}_{x_k}\{(X_1, \ldots, X_\ell) \in A\}, \quad (1.27)$$

Page 57, equation (4.39). This should be

$$\widehat{P}(n,n) = \widehat{P}(n, n-1) = 1/2. \quad (4.39)$$

Page 90, 14d [R. Pinsky, 9/3/09]. Replace n^2 by n^d. Thus displayed equation below it should read

$$\Phi_\star \le \Phi(V_1) = \frac{2(2d)}{2d(n^d+1)} \le 2n^{-d}\,.$$

Page 101, 2d [R. Pinsky, 9/3/09]. The displayed equation should be

$$(a_1 a_2 \ldots a_m) = (a_1 a_2)(a_2 a_3) \cdots (a_{m-1} a_m)\,.$$

Page 117, 1u [R. Pinsky, 9/3/09]. For notationaly consistency, replace $c(x,y)$ by $c(\vec{xy})$.

Page 120, 2d. The identity should read "$I(\overrightarrow{v_1 v_2}) := I(\overrightarrow{v_1 v}) + I(\overrightarrow{v v_2})$".

Page 120, 5d [R. Pinsky, 9/3/09]. Replace "potentials" by "voltages".

Page 129, 5u [R. Pinsky, 9/3/09]. The term +1 should be +2.

Page 133, Proposition 10.13 [R. Pinsky, 9/3/09]. The upper bound when $d \geq 3$ is not proven. The effective between opposing corners of the d-dimensional box can be estimated via a multi-type Pólya's urn process, similar to the case $d = 2$. Details for the case $d = 3$ can be found in the paper "Pólya's theorem on random walks via Pólya's urn", by D.A. Levin and Y. Peres, to appear in *American Mathematical Monthly*; also available at `http://www.uoregon.edu/~dlevin/polya.pdf`.

Page 139, Corollary 10.22 [R. Pinsky, 9/3/09]. The statement should be for $d = 2$. The lower bound is not proven in the text, and should be omitted from the statement.

Page 133, Equation (10.17) [3/1/09]. This should be

$$2\mathbf{E}_a(\tau_b) = n^d \mathcal{R}(a \leftrightarrow b). \tag{10.17}$$

Page 134, Remark 10.15, last line [1/11/2009]. The phrase "time to" should be "probability at"

Page 140, Exercise 10.9, line 3. $\sum_{k=0}^{\infty} b_k s^l$ should be $\sum_{k=0}^{\infty} b_k s^k$.

Page 141, Notes, line 1. This sentence should be moved to the Notes of the next chapter.

Page 197, Theorem 14.12 [A. Adelmann, 3/9/09].

In the statement of the theorem, (14.21) should be the following quantity:

$$2n \left\lceil \frac{n \log(n) + n \log(3nq^n/\varepsilon)}{c(q, \Delta)} \right\rceil \left\lceil \frac{27qn}{\eta\varepsilon^2} \right\rceil . \tag{14.21}$$

Replace the displayed equation before (14.22) by

$$t(n, \varepsilon) = \left\lceil \frac{n \log(n) + n \log(3nq^n/\varepsilon)}{c(q, \Delta)} \right\rceil \geq t_{\text{mix}} \left(\frac{\varepsilon}{3nq^n} \right)$$

and replace (14.22) by

$$\|P^{t(n,\varepsilon)}(x_0, \cdot) - \pi_k\|_{\text{TV}} \leq \frac{\varepsilon}{3nq^n} . \tag{14.22}$$

Replace the right-most quantity in the displayed equation below (12.22) and in (14.23) by $\frac{\varepsilon}{3nq^n}$. Replace the right-most quantities in the displayed equation above (14.26) by $n \cdot \frac{\varepsilon}{3nq^n} = \frac{\varepsilon}{3q^n}$. Replace (14.26) by

$$\mathbf{E}(W) = \frac{1}{|\Omega|} + \tilde{\varepsilon}, \quad \text{where } |\tilde{\varepsilon}| \leq \frac{\varepsilon}{3q^n} . \tag{14.26}$$

Replace the end of the proof following the sentence that begins "By Chebyshev's..." with:

Therefore

$$\mathbf{P}\left\{\left(1-\frac{\varepsilon}{3}\right)\mathbf{E}W \le W \le \left(1+\frac{\varepsilon}{3}\right)\mathbf{E}W\right\} \ge 1-\eta,$$

and since $\mathbf{E}W = 1/|\Omega| + \tilde{\varepsilon}$,

$$\mathbf{P}\left\{\left(1-\frac{\varepsilon}{3}\right)\left(\frac{1}{|\Omega|}+\tilde{\varepsilon}\right) \le W \le \left(1+\frac{\varepsilon}{3}\right)\left(\frac{1}{|\Omega|}+\tilde{\varepsilon}\right)\right\} \ge 1-\eta.$$

Applying $|\tilde{\varepsilon}| \le \frac{\varepsilon}{3q^n} \le \frac{\varepsilon}{3|\Omega|}$ shows that

$$\mathbf{P}\left\{\left(1-\frac{\varepsilon}{3}\right)\frac{1}{|\Omega|} - \frac{\varepsilon}{3}\frac{1}{|\Omega|} + \frac{\varepsilon^2}{9}\frac{1}{|\Omega|} \le W \le \left(1+\frac{\varepsilon}{3}\right)\frac{1}{|\Omega|} + \frac{\varepsilon}{3}\frac{1}{|\Omega|} + \frac{\varepsilon^2}{9}\frac{1}{|\Omega|}\right\} \ge 1-\eta.$$

Thus

$$\mathbf{P}\left\{(1-\varepsilon)\frac{1}{|\Omega|} \le W \le (1+\varepsilon)\frac{1}{|\Omega|}\right\} \ge 1-\eta.$$

For every step of the Glauber dynamics we need two uniform random variables (one for the vertex and one for the colour). We need a_n samples for every Ω_k, which shows that at most (14.21) uniform variables are required.

Page 241, line 2, Proposition 17.20 [E. Lubetzky, 12/24/09]. Proposition 17.20 Condition (ii) should be $Z_{t+1} - Z_t \le B$ without any conditional expectations.

Page 300, Question 7 [A. Nachmias, 4/7/09]. The condition should be added that the graph have spectral gap bounded away from zero.