MATH 241, LECTURE 19

1. Stationary and singular points

Definition 1. *A number x in the domain of a function $f(x)$ is a stationary number if $f'(x) = 0$ and is a singular number if the derivative at x does not exist.*

Example 2. *Find all the stationary points for the function $f(x) = x^3 - \frac{9}{2}x^2 + 2x - 5$. Find all the singular points of the function $g(x) = |5 + 4x - x^2|$. Graph these functions and say what you see at critical points.*

And we mentioned last time that finding stationary and singular points was the key to understanding where a function had relatively high or low values.

Example 3. *Find where the following functions are increasing or decreasing, then find the stationary points and graph the function near those stationary points.*

- $-\frac{1}{3}x^3 + 4x - 27$.
- $e^x - x$.
- $\frac{e^x}{x-2}$

1.1. Relative extrema.

We see that if a function changes from increasing to decreasing at a stationary or singular point, that function obtains some kind of maximum (there is a similar statement of minima, which you should think about). We now formalize this.

Definition 4. *Let $P = (c, f(c))$ be a critical point of a function $f(x)$.*

- *P is a relative maximum if $f'(x) > 0$ for nearby $x < c$ and $f'(x) < 0$ for nearby $x > c$.*
- *P is a relative minimum if $f'(x) < 0$ for nearby $x < c$ and $f'(x) > 0$ for nearby $x > c$.*
- *P is not a relative extremum if $f'(x)$ has the same sign on both sides of c.*

Example 5. *Find the relative maxima and minima of the functions from the previous example.*

If we are looking for largest or smallest values of a function, it often depends on where the function is defined. In particular, a relative maximum or minimum can also occur on an endpoint of the domain. For example, you were shortest at the moment you were born, so time $t = 0$ is where your height function has a minimum.

We now include endpoints of the domain whenever we refer to critical points (because it is critical to check for maxima and minima at those points!).

Definition 6. *If f is defined only over an interval from a to b, we say that f has a relative minimum at a if $f'(x)$ is positive for values of x close to a. We say that ... (you fill in the rest!).*

Example 7. *Classify the critical points (as relative maxima, relative minima, or neither) of $\frac{x^2-3}{e^x}$ over the interval from -4 to 4. Check the answer with the graph.*

Example 8. *Classify the critical points of the function $\ln(|x^2 - 2| + 1)$ over the interval from $-e$ to e.*

Example 9. *The revenue from Pet Rocks was*

$$R(t) = \frac{63t - t^2}{t^2 + 63},$$

where revenue is measured in millions of dollars and time is measured in weeks after June 5, 1967. When is the revenue at its maximum? What is that maximum?