MATH 241, LECTURE 24

1. Some problems

We have spent the term setting up the machinery of calculus, learning how to calculate derivatives, and then applying derivatives in various settings. Today we will review some of these main applications - optimization, understanding graphs of functions, and related rates. On Friday we will focus on a more real-world application.

Example 1. *At ACME Anvils, output is $Q = 60K^{\frac{1}{3}}L^{\frac{2}{3}}$ where K is the capital inverstment (in thousands of dollars) and L is the size of the labor force, measured in worker-hours. If output is kept constant, at what rate must capital investment change at a time when $K = 8$, $L = 1000$ and L is increasing at the rate of 25 worker-hours per week?*

Example 2. *Recall that the volume of a cylinder of height h and radius r is given by $\pi r^2 h$, and its surface area is $2\pi r^2 + 2\pi rh$. If a can is made of alloys which cost 5 cents per square centimeter on top and bottom but 3 cents per square centimeter on the sides, find the cheapest way to make a can which will hold 200 cubic centimeters.*

Example 3. *Sketch the graph of a function $f(x)$ with the following properties: the limit of $f(x)$ as $x \to \infty$ is -1 and as $x \to -\infty$ is 5; there are stationary points at $x = 0, 3$ and a singular point at $x = -2$. $f'(x)$ is negative for $x < -2$, positive between -2 and 0, negative between 0 and 3, and positive for $x > 3$; $f''(x)$ is negative for $x < -2$, positive between -2 and -1, negative between -1 and 2, positive between 2 and 6, and negative for $x > 6$.*

Example 4. *Maximize profit when the price at which q units can be sold is $p(q) = 25 - q$ and it costs \$10 to produce each unit.*

Example 5. *If one is consuming a refreshing malt beverage at a rate of $8t - t^2$ cubic centimeters per second, coming from a can with width 5 centimeters, how fast the level of liquid dropping after three seconds?*

Example 6. *Find where the following functions are increasing, decreasing, concave up, and concave down:* $\ln(3x^2 + 1)$, $\frac{x-3}{2x+5}$, e^{-x^2}.

Example 7. *The revenue from Pet Rocks was*

$$R(t) = \frac{63t - t^2}{t^2 + 63},$$

where revenue is measured in millions of dollars and time is measured in weeks after June 5, 1967. When is the revenue at its maximum? What is that maximum?