MATH 241, LECTURE 17

0.1. **Some practice with logarithms, exponential functions, and the chain rule.** Derivative rules are great fun because they can be combined. Taking the derivative of some functions requires every rule we know.

Example 1. *Take the derivatives of* $(\frac{e^x \ln x}{x^2+1})^9$ *and* $\ln(\frac{e^x}{\sqrt{x}})^9$.

1. Implicit Differentiation

At the beginning of the term, we discussed the usefulness of functions in describing many different relationships. But we also emphasized that not all relationships are described by functions. For example, in economics supply and demand are related, but one is not a function of the other. In mathematics, the equation $x^2 + y^2 = 25$ is one of the most basic; the points which satisfy it constitute a circle of radius 5.

Amazingly, we can follow rules for taking derivatives to find the rate of change of one variable with respect to another in some cases where there is a relationship between them which is not described by a function.

It is best to start with examples and then summarize the general method as and after we do them.

Example 2. *Find the equation of the tangent line to the circle* $x^2 + y^2 = 25$ *at the point* $(3, 4)$.

Here are the steps:

- Remember the point-slope form for a line, reducing problem to finding $\frac{dy}{dx}$.
- Differentiate the equation, treating y as a function of x even though it is not expressed as one. Remember to use the chain rule to differentiate y, since we are treating it as a function of x!
- The resulting expression has $\frac{dy}{dx}$ in it, since we have used the chain rule on y. Now do algebra to solve for it.
- Now that we know a formula for $\frac{dy}{dx}$, we can plug in values for x and y to find the slope of the tangent line and finish the problem.

Note that we could have done this problem by solving for y, namely $y = \sqrt{25 - x^2}$, and finding the slope by taking the derivative as we are used to. This is a good way to check that our answer is correct in the previous case.

We practice with a couple examples before trying to apply this technique to more complicated problems.

Example 3.

- *Find* $\frac{du}{dv}$ *if* u *and* v *are related by* $u^3v - 2u^2v^2 + v^4 = 17$.
- *Find* $\frac{dy}{dx}$ *when* $e^{xy+4x-3} = 5x^2$. *(Do this twice, the second time by taking the natural logarithm of both sides).*

Example 4. *Two manufacturers of widgets are in direct competition. Because of the many variables in pricing and publicity, the number of widgets they sell does not add up to a constant, but* $6x^2 + xy + 5y^2 = 120,000$*, where* x *is the number of widgets sold per day by the first company and* y *by the second. If both companies are currently selling* 100 *widgets, what would be the effect on the first company if the second is increase sales by ten widgets per day.*