MATH 241, LECTURE 23

1. Curve sketching

We can assemble which comes through our analysis of a function and its first and second derivatives in order to get a good picture of the graph of a function.

- Plot a few values of the function, including $f(0)$, which is where the function crosses the y-axis.
- Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$, if they exist, in order to see where the graph "begins" and "ends" and draw in these ends if they can be determined.
- Find and plot the places where the function crosses the x-axis by solving the equation $f(x) = 0$.
- Calculate the first derivative and find where it is zero, positive positive and negative in order to find the critical points of f and determine where f is increasing and decreasing.
- Calculate the second derivative and find where it is zero, positive and negative in order to determine where f is concave upward and downward.
- Check where the sign of the derivative changes or use the second derivative test to determine which critical points are local maxima or minima.
- Draw in "cups" at local minima, "caps" at local maxima, and one of four kinds of curve, as sketched on the board, in the regions where the the signs of the first and second derivative do not change.
- Fill in the parts of the graph in between the curves you have put in.

Example 1. *Sketch the graph of $f(x) = 2x^3 + 3x^2 - 12x - 7$, and of xe^{1-x}.*

2. Related rates

In some problems all quantities in question are functions of some variable (most often time) but this dependence is not explicit; only a relationship is known. The techniques we have used for implicit differentiation can be used to find the derivative of one quantity if we know the derivative of the other.

The classic related rates problem, one used to torture calculus students for as long as the subject has been taught, is the problem of the ladder.

Try not to let this problem and its cousins make its way into your nightmares.

Example 2. *A ladder, which is ten feet long, is leaning against a wall. Its feet begin to slide out from under it, and its top falls at a constant rate of one foot per second. How fast is the foot of the ladder moving when the top of the latter is at 8 feet? What about 4 feet?*

The notion of related rates, like that of implicit differentiation, is based on the fact that taking the derivative of both sides of a valid equation gives rise to a valid equation (as long as the variable with respect to the derivative is being taken is clear, and that the chain rule is applied properly).

Example 3. *At ACME Anvils, output is $Q = 60K^{\frac{1}{3}}L^{\frac{2}{3}}$ where K is the capital inverstment (in thousands of dollars) and L is the size of the labor force, measured in worker-hours. If output is kept constant, at what rate must capital investment change at a time when $K = 8$, $L = 1000$ and L is increasing at the rate of 25 worker-hours per week?*