MATH 251, LECTURE 3

1. The ins and outs of linear functions

There are many ways to describe a given linear function. An important skill to develop is the ability to translate between the different descriptions. Key to many of these descriptions is the notion of slope.

Definition 1. *The slope of the line $y = mx + b$ is equal to m. It measures the change in y if x is increased by one. If one is not given the slope explicitly, it can be computed by $m = \frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are any two points on the line.*

Example 2. *Sketch some lines with slope 1, -1, 2, -3, $\frac{1}{2}$ and $\frac{-2}{3}$.*

Example 3. *The points $(-1, -1)$ and $(3, 7)$ are both on the line $y = 2x + 1$. We can verify the formula for m in this case, and then take two other points on the line and use them to calculate the slope.*

The different descriptions we will use are as follows, listed from simplest to most complicated. We translate each to the standard form, and give an example.

- Standard, slope-intercept form: $y = mx + b$.
- Linear equation form: $ax + by = c$.
- Point-slope form: "the line which goes through (x_0, y_0) with slope m" namely, $(y - y_0) = m(x - x_0)$.
- Two-point form: "the line between (x_1, y_1) and (x_2, y_2)," namely $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

Example 4. *Find the equations - in slope-intercept form - of the following lines:*

- *The line $3x + 4y = 7$.*
- *The line through the points $(-1, 4)$ and $(2, 3)$.*
- *The line through the point $(1, 2)$ with slope 3.*

Two lines are parallel if they have the same slope.

Example 5. *Find the line with y-intercept equal to three which is parallel to the line through the points $(2, 4)$ and $(-1, 2)$. Graph both lines.*

Two lines are perpendicular if they intersect at ninety degree angles. In equations, it means that if the slope of one line is m, the slope of the other is $-m$.

Question: Why does this make sense?

Example 6. *Find the equation of the line perpendicular to the line $2x - 3y = 3$ at the point $(3, 1)$.*

2. Quadratic functions

After linear functions, the next simplest kinds of functions are quadratic functions. Our first applications of ideas from calculus will often be to quadratic functions. But we will review basic properties of these functions now, so they are familiar when we come to them later.

Definition 7. *A* quadratic function *is of the form $f(x) = ax^2 + bx + c$, where a, b and c are fixed numbers and $a \neq 0$. A quadratic function has a* parabola *as its graph.*

Question: What happens if $a = 0$?

A parabola either "opens up", meaning the function first decreases and then increases (going from left to right on its graph). Or the parabola "opens down", which means...

Question 8. *What about a parabola's defining equation determines whether it opens up or down?*

Theorem 9. *A parabola intersects the y-axis at the point $(0, c)$. A parabola intersects the x-axis at points given by the quadratic equation. A parabola reaches its maximum or minimum when $x = \frac{-b}{2a}$.*

We can easily verify the first two statements. The third statement can also be verified through the algebra of "completing the square", but will be very easy to see with a little calculus.

Example 10. *Graph the function $f(x) = -\frac{1}{2}x^2 + 3x - 4$, indicating the places where the graph crosses the axes, as well as the minimum or maximum.*

Example 11. *Suppose the total profit made by manufacturing x thousands of units of Freshies doggie biscuits is given by $P(x) = -0.4x^2 + 80x - 20$ (measured in tens of thousands of dollars).*

- *Explain why this is a reasonably-shaped profit function. For example, $P(0) = -20$; does that make sense?*
- *Find the optimum number of Freshies to produce.*