MATH 241: CALCULUS FOR BUSINESS AND THE SOCIAL SCIENCES, LECTURE 1

1. What to expect:

- Lecture classes MWF 2, Mackenzie 240A.
- Review sections on M or Tu. Quiz most weeks at the beginning of class time.
- Homeworks due every Wednesday at the beginning of class (please come early).
- Two exams, one final.

The syllabus and all readings and assignments are at:
`http://noether.uoregon.edu/~dps/241/`

The choice of material will, as advertised, be geared towards business, social science, and similar majors. Economics majors should be aware that they need a more advanced grounding in calculus (plus some other mathematics beyond calculus) if they want to go to graduate school.

1.1. **Lecture classes.** We will have combined projector and blackboard presentations. PDF files of the overheads will be placed on the web. It is strongly suggested that you print them out and bring them to class, and then fill in what is done on the blackboard. The notes are to provide the organizational framework for the material, but are not intended to be complete.

Class participation during lecture is strongly encouraged.

2. Why calculus?

- It is one of the greatest creations of the human mind. Without it one could not: understand the motion of the heavens and send a man to the moon; understand electro-magnetism and build a computer; understand the atom and build an MRI machine; understand modern economics or finance and be able to set monetary policy. It is part of our culture.
- It is weight training for the mind, perhaps one of your greatest challenges academically. In particular, it develops logic skills (as desired, for example, by law schools).
- It is beautiful in both the big picture and in the details and in how they fit together, like an amazing piece of music or poem or car.

3. Difficulties students have with calculus (really, with math in general).

- Allowing enough time to make mistakes and figure out what went wrong.
- Figuring out the right tool for the job.
- Algebra.

One important tip to improve your algebra is learning to substitute numbers into algebraic expressions.

Example 1. $(x+y)(x-y) \stackrel{?}{=} x^2 - y^2$
Substitute $x = 3$, $y = 5$*:*
$(x+y)(x-y) = (3+5)(3-5) = 8(-2) = -16.$
$x^2 - y^2 = 3^2 - 5^2 = 9 - 25 = -16.$

These two expressions are equal. What does that mean? We suspect that this equality always holds. You should try to substitute three different sets of values in an equality you do not understand.

This example, and even a hundred more like it, does not imply that $(x+y)(x-y) = x^2 - y^2$ for all x and y. To establish it once and for all, we must apply the rules of algebra.

$$\begin{aligned}(x+y)(x-y) &= (x+y)x - (x+y)y \\ &= x^2 + yx - [xy + y^2] \\ &= x^2 + xy - xy - y^2 \\ &= x^2 - y^2\end{aligned}$$

More examples.

Example 2. $a^{x+y} = (a^x + a^y)$ *or* $a^x a^y$*?*

Example 3. $\frac{x^2-y^2}{x-y} \stackrel{?}{=} x + y$.

4. Zeroth topic in calculus: functions

Calculus is the study of how functions change and aggregate.

But what is a function?

Informally, a function represents a relationship between two (sets of) quantities in which the first quantities uniquely determine the second.

4.1. **Function, or not a function?** Which of the following relationships can be represented by a function?

- There is a correlation between a person's height and weight.
- At any given time, one can measure the temperature at the corner of 13th and University, or the value of the S&P 500, or the position of a ball which was thrown or your current GPA.
- The taxes one pays are determined by one's income and deductions.
- The measures of the three angles of a triangle add up to 180 degrees.
- There is a relationship between supply and demand in economics.
- Population density can be measured for each square mile of land in the United States.
- All items in the store are 30% off.
- The cost of an airline ticket is a function of many variables.

Numbers can represent many different real-world phenomena, but the same rules apply to numbers regardless of what they represent. We will be learning about rules which apply to any kind of function.

Sometimes, what we do will be abstract, but just as for numbers, abstraction will give wide applicability.

Definition 4. *A function assigns to each input, from some set called the* domain *of the function, a unique output, in a set called the* range *of the function.*

Examples can be produced from our list above.

Throughout this course, the domain and range of a function will usually be some collection of numbers and a function is most often denoted by "$f(x) = \cdots$". Informally, we think of the domain as "numbers which can go in" and the range as "numbers which can come out". Sometimes the "same" function can have different domains and ranges.

Example 5. *What are the values at $x = 3$ and the possible domains and ranges for the following functions:*

- $f(x) = x^2$
- $f(x) = \sqrt{x+4}$
- $f(x) = \frac{x+5}{1-x}$