MATH 241, LECTURE 21

1. Applied optimization problems

First, we should remember how to find the largest and/or smallest values of a function over some interval.

- Find all stationary points (by finding the derivative and setting it to zero).
- Find all singular points.
- Find the endpoints (these are often given).
- Compared values at all of these points - the largest is the max and the smallest is the min.

In applied problems one usually has to work in order to find a single function to maximize. One needs to keep track of relationships and constraints.

Example 1. *Suppose advertising costs $1000 per unit (say for magazine adds), and product development costs $20000 per unit. Suppose that the profits generated from x units of advertising and y units of product development are xy^2 thousands of dollars. If a company has $10000 to spend on advertising and product development together, how should the money be allocated in order to maximize profits?*

Applied problems are not as "cut and dried" as other exercises, but there are some elements common to most such problems.

- Identify all variables involved. They often need to be named.
- Identify which variable needs to be maximized or minimized.
- Identify all relationships between and constraints on variables.
- (Optional) Restate problem abstractly in the form "find the maximum of the function ... with the constraint(s) that ..."
- Use the relationships to express one variable as a function of one other variable.
- Use our optimization techniques above to find the minimum or maximum of the appropriate variable.

Example 2. *Farmer Fred has 100ft of fencing to use to enclose his sheep in a rectangular area next to a river. What is the largest area which he can enclose?*

Example 3. *Suppose the top and bottom of a box is made of a metal which costs 10 cents per square centimeter and the sides are made of a metal which costs 12 cents per square centimeter. What is the largest volume can which can be made from two dollars of material?*

Example 4. *Suppose the daily production level at a factory is modeled by a Cobb-Douglas production function $P = L^{0.7}C^{0.3}$, where L is the number of workers and C is the cost of materials measured in thousands of dollars. If each worker costs the company $200 per day, what is the maximum production level which can be achieved with a total cost of $10000 per day.*