MATH 242, LECTURE 17

1. Linear programming in applied problems

Linear programming, and our steps for finding maxima and minima, are applicable in a number of kinds of applied optimization problems. As we practice these problems, we will see that setting them up accurately can be more difficult than solving the mathematics problem which arises, which for bounded regions is pretty straightforward given the steps we outlined last lecture.

Example 1. *You want to invest up to \$10,000 in the stock market. Dorf shares sell for \$50 each, yield a dividend of 5% and have a risk index of 2.0 each. Cubik shares sell for \$25 each, yield a dividend of 8% and have a risk index of 3.0 each. How many shares of each stock should you buy if you want your dividends to be at least \$500 and you want to minimize your total risk index?*

The mathematics of investments and finance can become highly complicated.

2. Unbounded feasibility regions

As stated when we gave the steps for solving linear programming problems, if the feasibility region is unbounded we need to check the values of the objective function at boundary points which "go to infinity". More formally, we systematically approximate the unbounded feasibility region by bounded feasibility regions which grow to fill the original (as is best seen in a picture).

Example 2. *Find the maxima and minima, if they exist, of the function $F(x, y) = 2x + 4y$ over the constraint region* $\begin{cases} x + y \geq 2 \\ y \geq 0 \\ -x + 2y \leq 4 \end{cases}$ *. Repeat the problem for the function $F(x, y) = 4x + 4y$.*

In the second half of this example, we see that an optimum value can occur along all of the boundary points in a boundary line when, the boundary line is parallel to the lines of constant value for the objective function.

Example 3. *Suppose that in a production facility the proportion of two components is never allowed to be more than 2 to 1, so they are always ordered together in less than 2 to 1 proportions to one another. Suppose the first component costs \$20 each and the second \$30 each, and orders must be at least \$500. What is the minimum number of components which can be purchased?*