MATH 242, LECTURE 2

1. Antidifferentiation: the indefinite integral

Mathematics is full of constructions which, once they're understood and found useful to be done, are also found useful to be undone. For example, subtraction undoes addition and division undoes multiplication. So it should come as no surprise that it can be very useful to undo the derivative.

Definition 1. *We say that a function F is an anti-derivative of f if the derivative of F is f.*

Example 2. *The function x^2 is an anti-derivative of $2x$. The function $x^2 + 7$ is also an anti-derivative of $2x$. The function e^{x^2} is an anti-derivative of $2xe^{x^2}$.*

Example 3. *Name anti-derivatives of $3x^2$ and $\frac{1}{x}$.*

Notice that in our first two examples, one function, namely $2x$ had two anti-derivatives, namely x^2 and $x^2 + 7$. In fact, any function will have many anti-derivatives, which makes taking anti-derivatives different in character from taking derivatives or doing algebraic manipulations.

Finding anti-derivatives is at first a process of trial and error. Since we know how to take derivatives, we can often guess what an anti-derivative might be, check if our guess is correct by taking its derivative, and then "fiddling around" to get the answer just right.

Example 4. *Find anti-derivatives for $x^2 + 5$, $\frac{1}{x}$, and e^{5x}.*

1.1. **Integral notation.**

We saw above that more than one function can be an anti-derivative for a given function. That may be worrisome at first, but the following theorem puts the situation under control.

Theorem 5. *If $F(x)$ is an anti-derivative for $f(x)$ then any other anti-derivative is equal to $F(x) + C$, where C is some constant (function).*

For example, we saw that x^2+7 is an anti-derivative for $2x$. And so is x^2, which is equal to $(x^2+7)+-7$. Another anti-derivative is $x^2 + 52$, which is $(x^2 + 7) + 45$. It feels more natural to say that any of these anti-derivatives is of the form $x^2 + C$, where C can be any constant.

To make matters more or less confusing (depending on your point of view), the collection of anti-derivatives of a function has another name.

Definition 6. *The family of all anti-derivatives of a function $f(x)$ is denoted $\int f(x)dx$, which is also called the indefinite integral. If $F(x)$ is some anti-derivative of $f(x)$, we have the equality (of families of functions) $\int f(x)dx = F(x) + C$.*

We will see later why the word "indefinite". This notation is named as follows: $\int$ is the integral sign; $f(x)$ is the integrand; dx denotes the variable of integration; and C is called the constant of integration.

Example 7. *Evaluate: $\int x^{-5}dx$ and $\int e^{5x}dx$.*

2. First applications of anti-derivatives

Anti-derivatives let us recover quantities from their derivatives - for example total cost from marginal cost or distance travelled from velocity.

Example 8. *The marginal cost for making Chiapets is $3q^2 - 48q + 320$ cents for the qth unit. The set-up cost for making the first Chiapet is \$20.00. What is the total cost for producing ten Chiapets?*

Example 9. *Basic physics dictates that, neglecting air speed, the downward acceleration of an object dropped due to gravity is constant,* 32 *feet per second per second. Find a formula for the distance travelled by an object dropped (not thrown, so it starts with no speed) over* t *seconds. How long would it take an object to fall 5000 feet (for example dropping it over the edge of the Grand Canyon)?*

These examples are the beginning of the subject of *differential equations*, which is the study of how to solve equations involving derivatives. All of the laws of basic, classical physics are written in terms of differential equations.