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Example 1. *You and your housemates have a two-bathroom house, but only one hot shower can be taken at a time. You have one housemate whose first class is at the same time as yours, and unfortunately you both like to take ten-minutes showers and start your day in the same half-hour window.*

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• *A random variable* X *represents the outcome of a*

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X	2	3	4	5	6	7	8	
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The example above is a discrete random variable - why?

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In this example X is called uniformly distributed between 0 and 3.

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2. If X *is distributed according to a distribution* D *then* $P(a \leq X \leq b)$ *is the area between the line* $x = a$*, the line* $x = b$*, the graph of* D*, and the* x*-axis.*

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$$D = \begin{cases} \frac{1}{12} & 1 \leq x \leq 3 \\ \frac{1}{2} & 3 \leq x \leq 4 \\ \frac{1}{9} & 4 \leq x \leq 7 \\ 0 & \textit{otherwise} \end{cases}$$

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- *What is the state space?*
- *What is $P(2 \leq X \leq 3)$?*
- *Which is more likely, that X is between 3 and 4 or that*

X *is between* 5 *and* 7*?*

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