I gather, young man, that you wish to be a Member of Parliament. The first lesson that you must learn is, when I call for statistics about the rate of infant mortality, what I want is proof that fewer babies died when I was Prime Minister than when anyone else was Prime Minister. That is a political statistic.

–Winston Churchill

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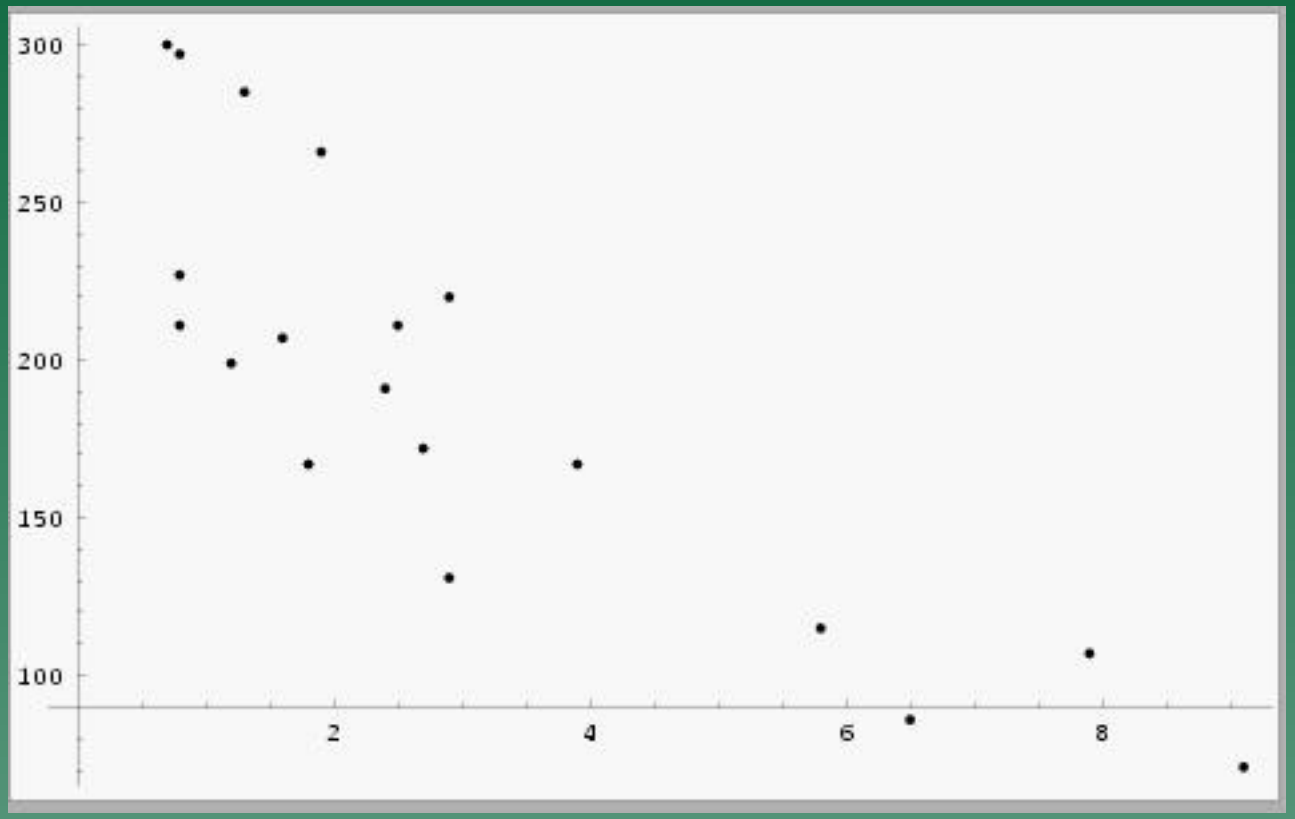
RESPONSE VARIABLE: *deaths per 100,000 from heart attack per year.*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The value of r is $-.843$ - how does that fit with what we see informally?

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$$b = r\frac{s_y}{s_x}$$

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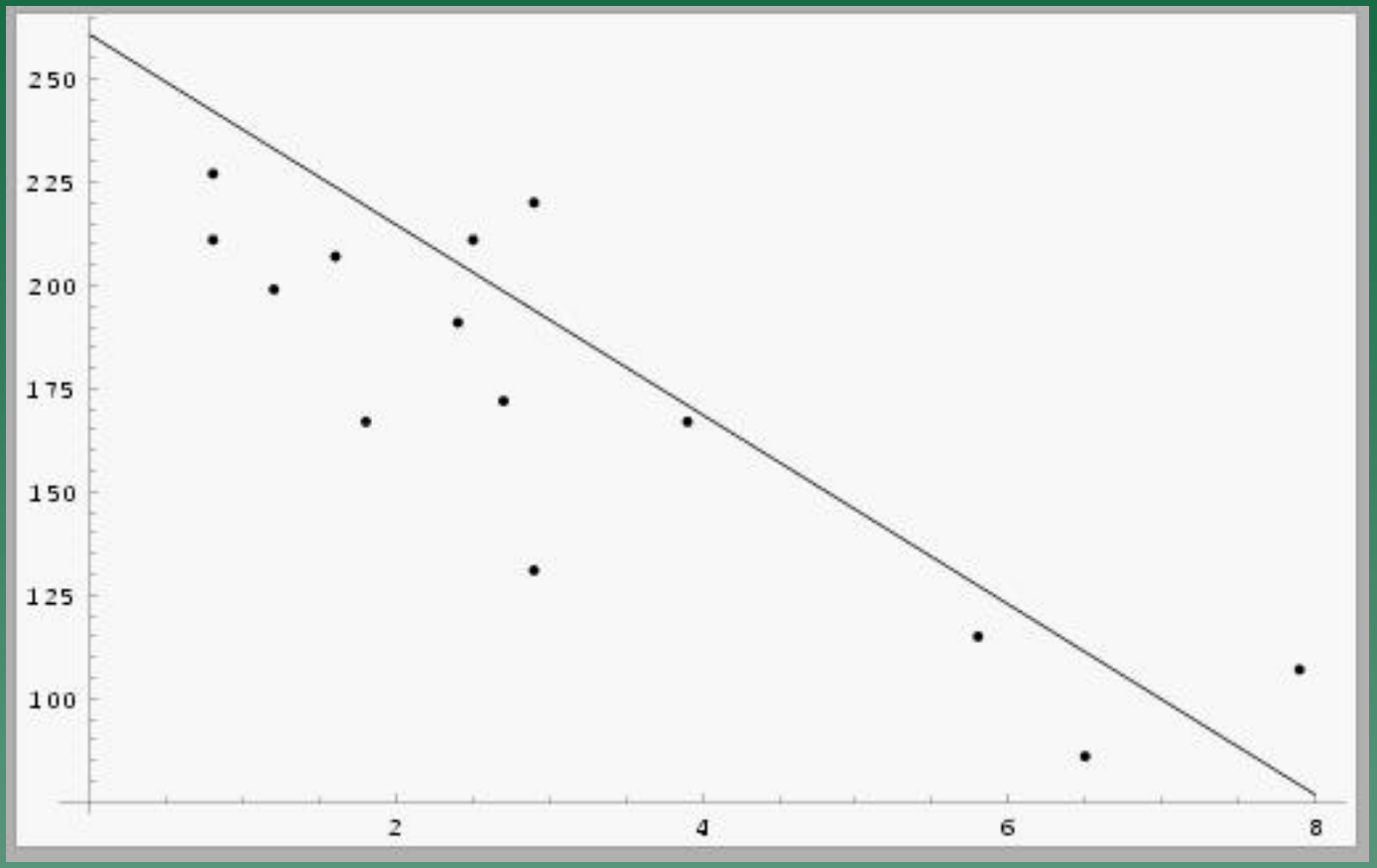
Let's plot the regression line over our scatterplot: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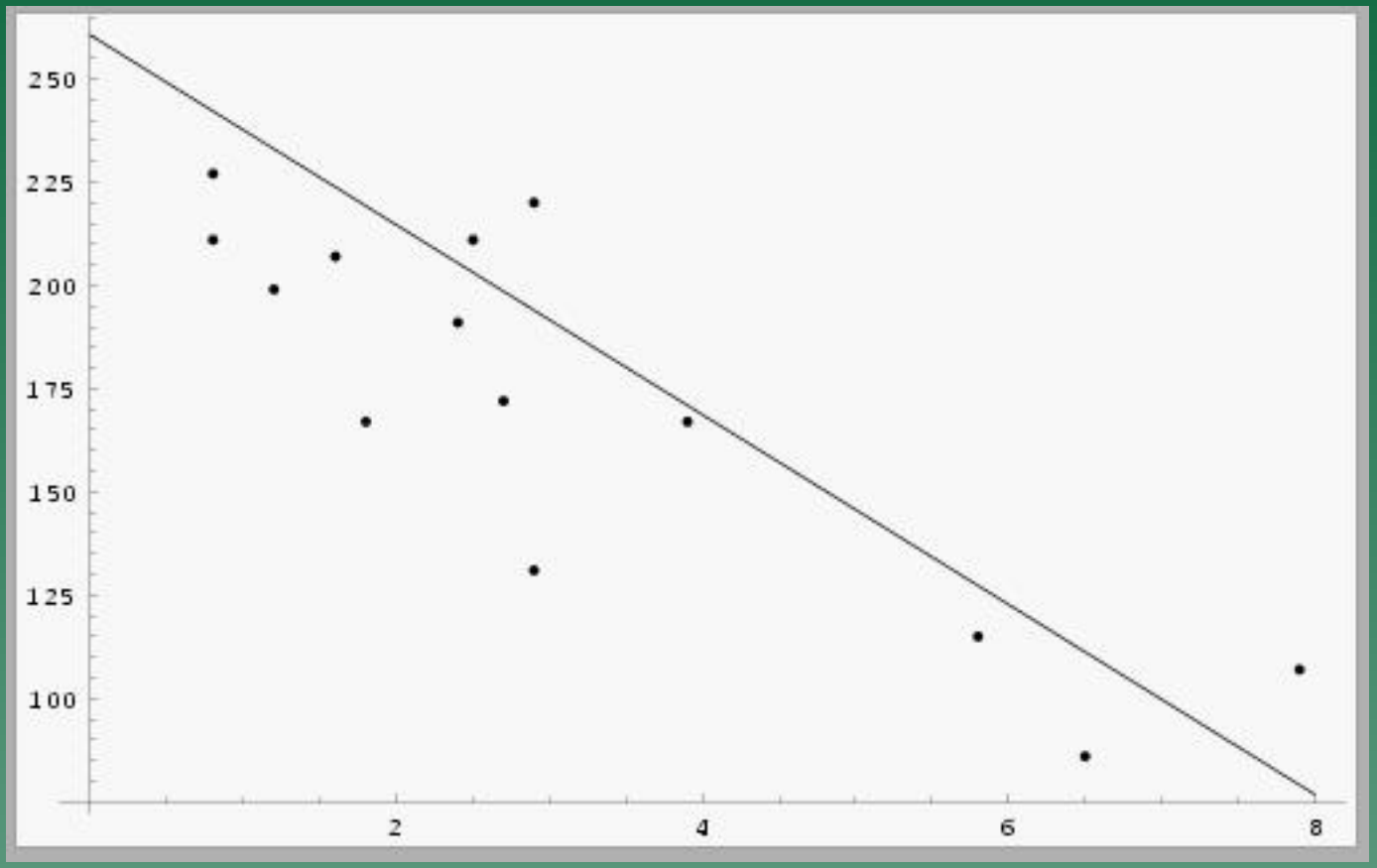

What do you notice?

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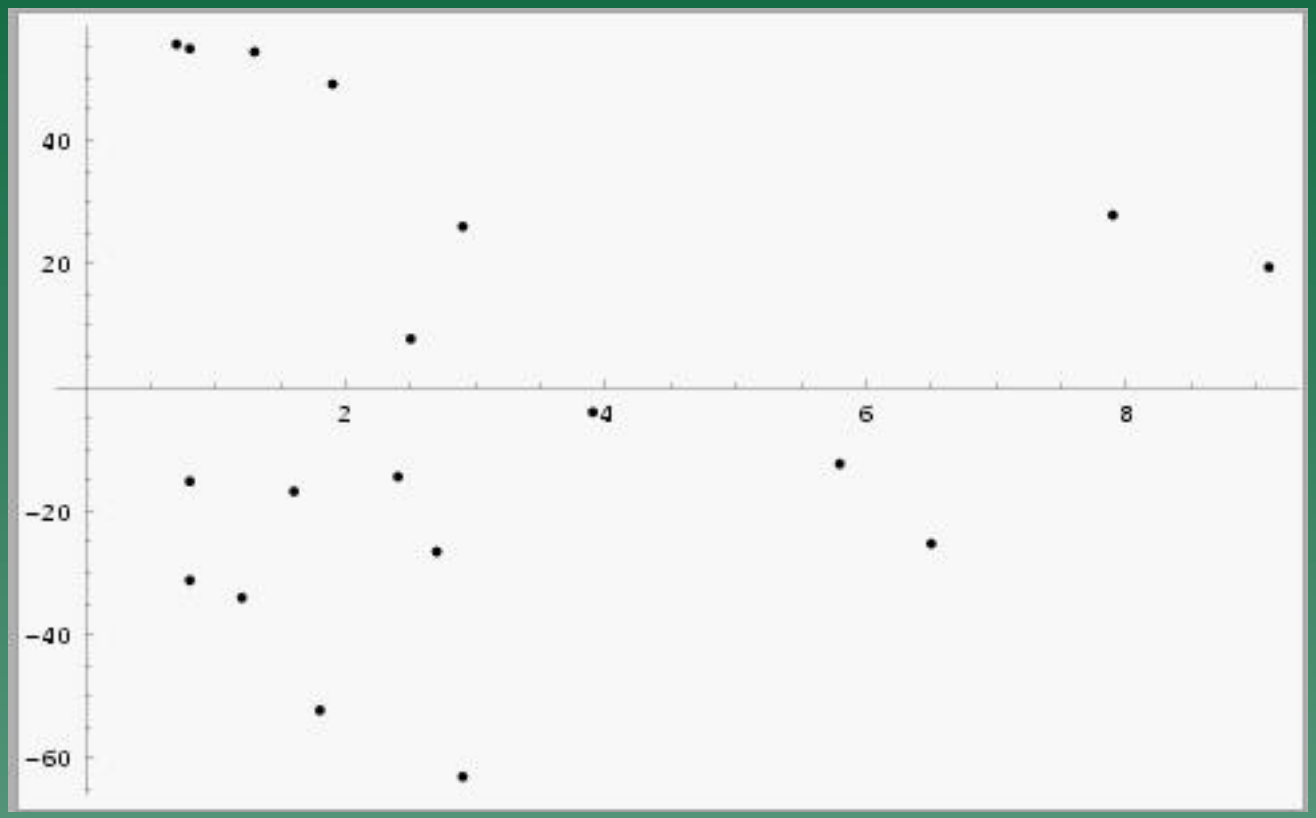
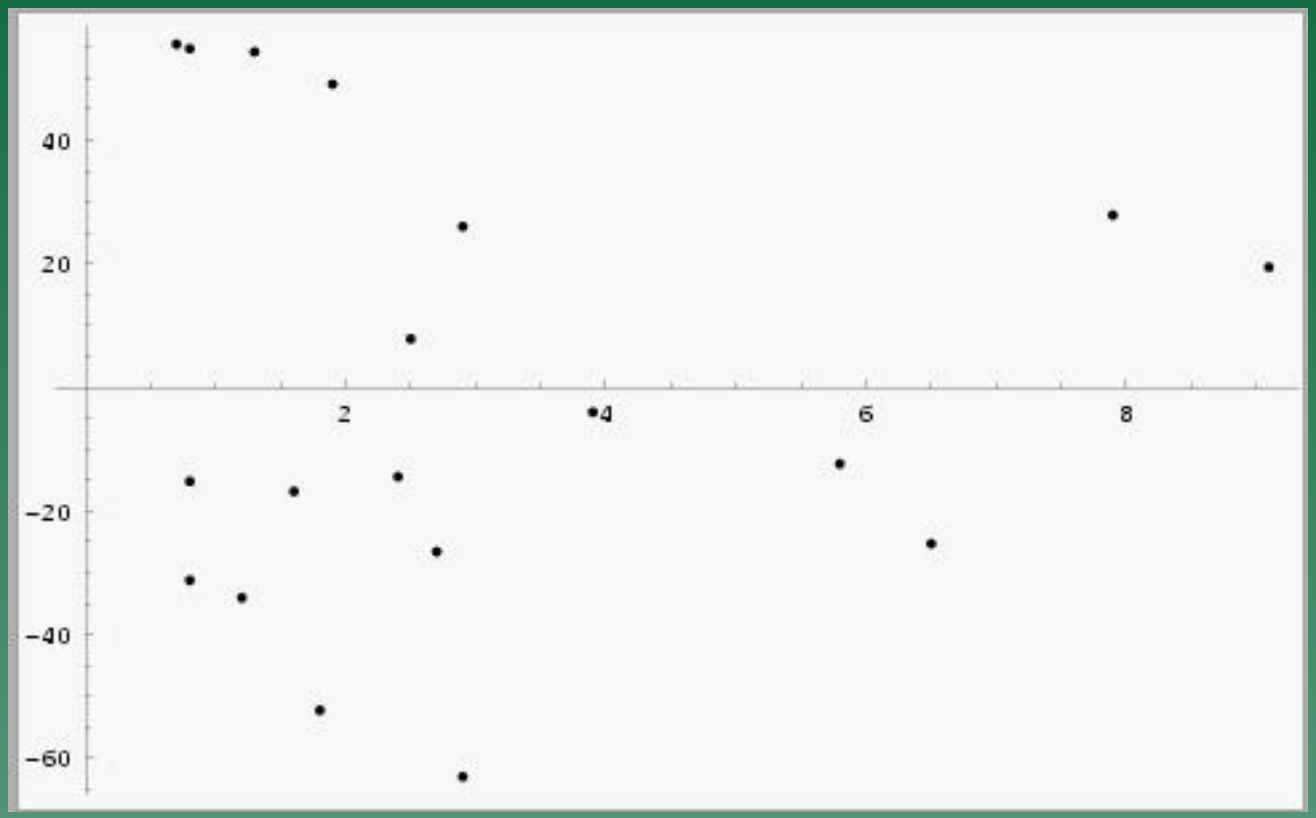
- *r^2 tells the fraction of the variation of y that is explained by the variation of x. (In our wine example, $r^2 = .71$.)*

Residual plot

We can make a residual plot by measuring how far each y value is from the predicted $\hat{y}$ value (for each x).



This gives a subjective impression of how well the regression line fits.



This gives a subjective impression of how well the regression line fits. If most y values are close to zero, the

fit is good.

Correlation and the regression line

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We take data from our dataset involving life expectancies (male, female and both), people per TV and people per doctor. We examine the variables "people per television" as a possible explanatory variable and "life expectancy" as a possible response variable.

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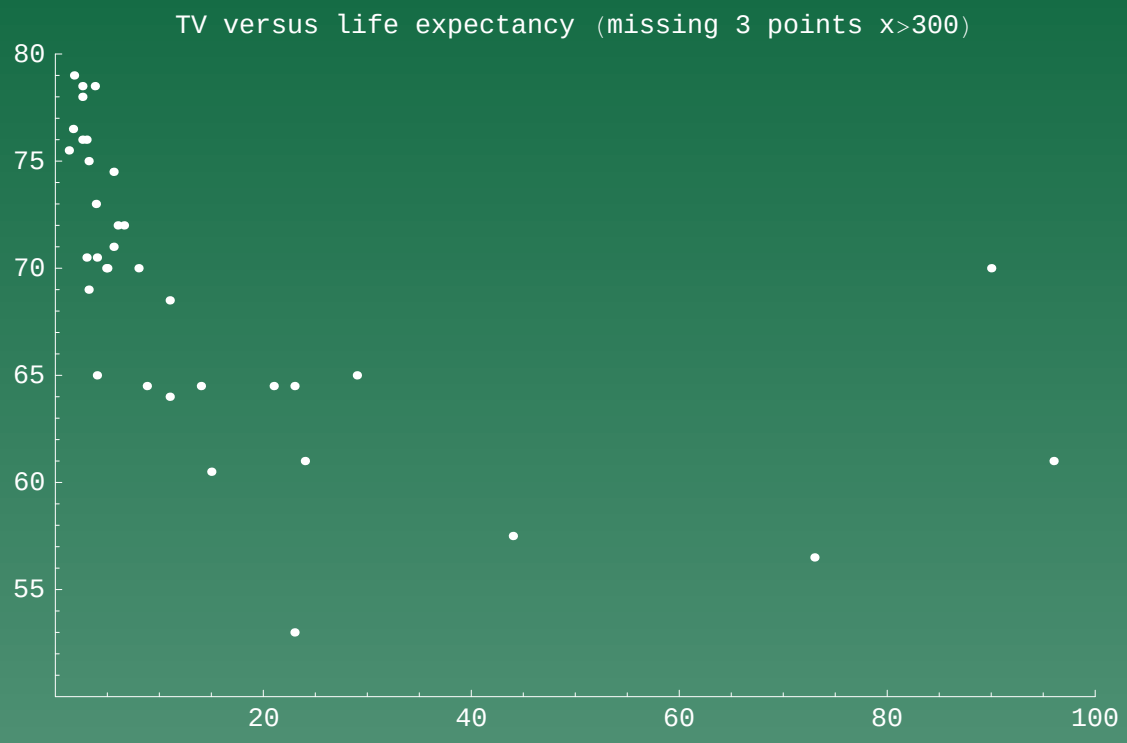
We'd like to use this data for two purposes.

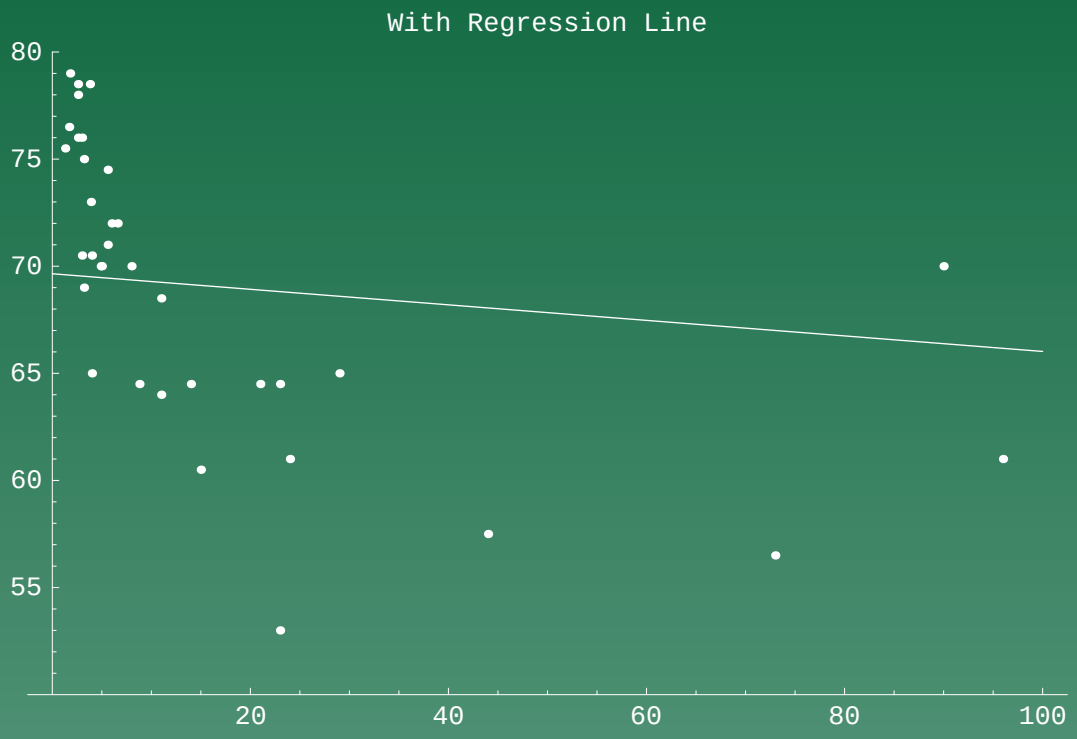
- Correlation does *not* imply causation. These two variable are reasonably strongly correlated, but obviously abundance of TVs does not cause a long lifespan.

- We'll calculate correlation and regression lines removing outliers to demonstrate that correlation and the regression line are *not* resistant.

We will take the opportunity to calculate the regression lines, and try to understand the numbers which go into them, along the way.

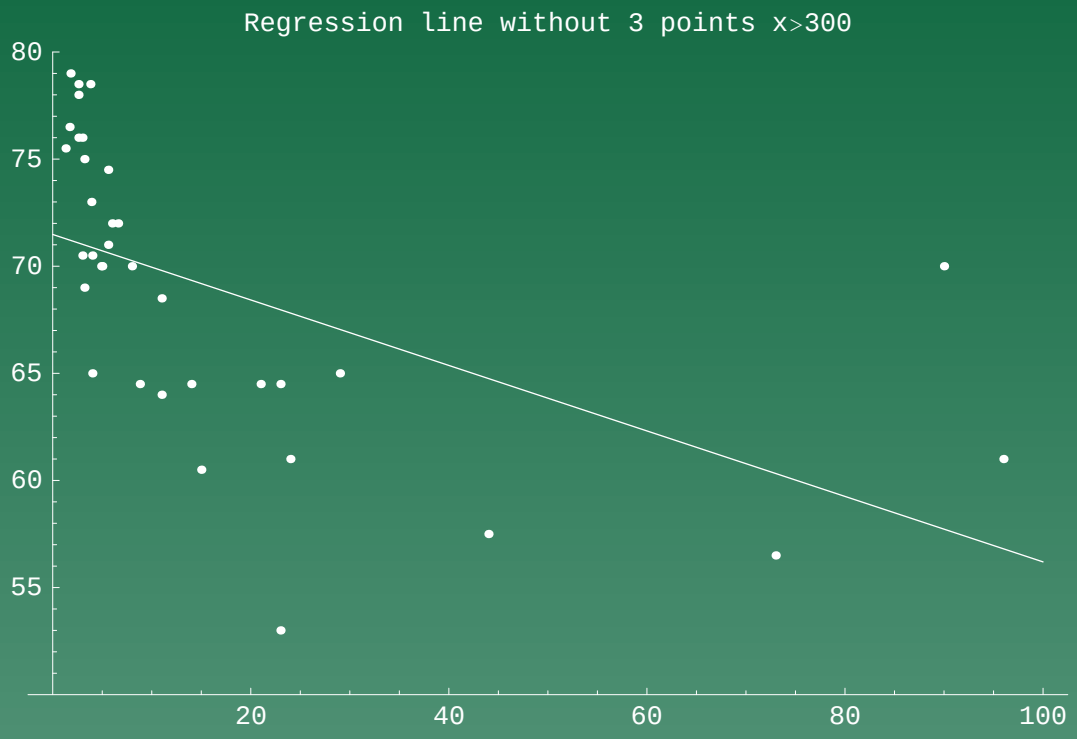


$$r = -.605847, r^2 = .36705$$



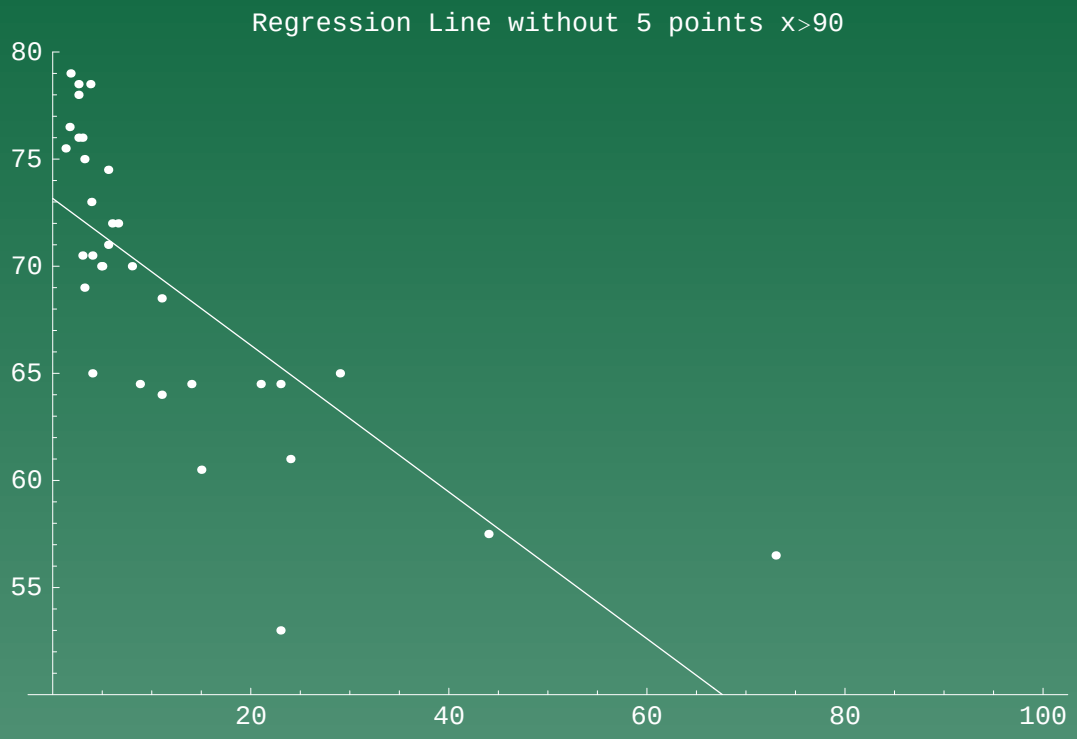
$$r = -.605847, r^2 = .36705$$

$$\hat{y} = 69.64 - 0.036x$$



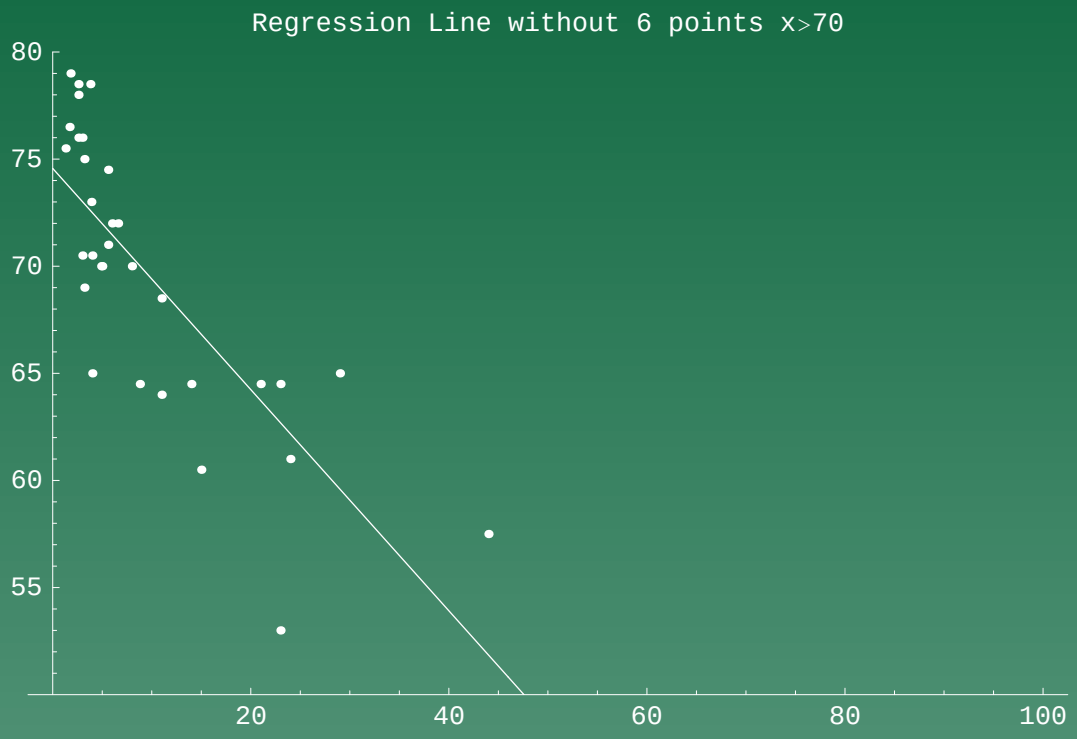
$$r = -.541398, r^2 = .293112$$

$$\hat{y} = 71.48 - 0.153x$$



$$r = -.740354, r^2 = .548123$$

$$\hat{y} = 73.16 - 0.343x$$



$$r = -.784158, r^2 = .614904$$

$$\hat{y} = 74.56 - 0.516x$$

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Beware the effects of outliers As we have illustrated with the TV vs. Life Span regressions. If a statistic can change a lot with the addition or subtraction of one data point, how heavily can we rely on it?

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