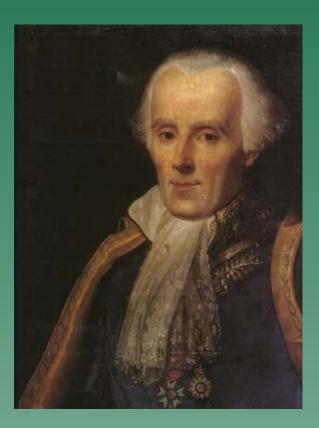
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The example above is a discrete random variable - why?

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In this example X is called uniformly distributed between 0 and 3.

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