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In order to find confidence intervals and test hypotheses, we need to understand standard deviations and errors. Fortunately some sharp mathematicians and statisticians come to our rescue.

Theorem 1. *If two distributions of size n_1 and n_2 have standard deviations σ_1 and σ_2, then the deviation for the observed difference is $\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$.*

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Theorem 2. *The following approximations may be used when t-procedures are applicable.*

- *With probability $C\%$ the (true) difference of means $\mu_1 - \mu_2$ has values between*

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where t^* is the critical value associated to the $t(n-1)$-distribution where n is the smallest of n_1 and n_2.

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To summarize, if we use the standard error SE in the places the single-sample standard error was used, we may use the same methods to understand the true difference of means from the observed difference of means.

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- *A sample of size 19 from population A, with mean* 54*, and sample standard deviation* 5*.*
- *A sample of size 23 from population B, with mean* 49 *and sample standard deviation* 4*.*

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Population	Mean	Sample Size	Sample mean	Sample s.c
Girls	μ_1	31	$\overline{x}_1 = 105.84$	$s_1 = 14.27$
Boys	μ_2	47	$\overline{x}_2 = 110.96$	$s_2 = 12.12$

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We wish to test this using our data.

2. We calculate our two-sample t-statistic

$$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{105.84 - 110.96}{\sqrt{6.569 + 3.125}} = \frac{-5.12}{3.114} = -1.644.$$

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   From the calculator, $P(t \leq -1.644) = .0553.$

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5. We can ask the calculator to *do* the test for us. This is under $\boxed{\text{STAT}}$, TESTS, 4:2-SampTTest. We get $df = 56.93$, $t = -1.64$, $P = .053$.

We *still* need to do step 4 (conclusion) above. And we need to do it carefully, because we've possibly lost track of what all our numbers mean.

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They took a sample of 51 patients. They assigned a group of 24 patients to receive "leech therapy," and a control group of 27 patients to receive a conventional pain-relief therapy (diclofenac gel).

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One week after the leech treatment (it was one treatment lasting a little over an hour involving 4 to 6 leeches), the leech group had a mean pain index of 19.3 with a standard

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Test the null hypothesis that the effect of treatment by leeches is the same as the effect of conventional treatments.