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One of the basic facts of probability and statistics is that if one takes larger and larger samples from a population, a statistic will approach its corresponding parameter (so for example $\overline{x}$ will approach μ). When formalized, this is called the "Law of Large Numbers." But this does not help us in our ultimate goal of extrapolating data help us in our ultimate goal of extrapolating data It says for example that we approach the right answer if we can survey larger and larger random samples of car buyers.

help us in our ultimate goal of extrapolating data It says for example that we approach the right answer if we can survey larger and larger random samples of car buyers. But if we can only sample 1000 people, then what we really want to know is how close we are to the actual parameter “in all likelihood.”

Sampling distributions

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We go thoroughly through an example looking at sampling distributions and comment thoroughly as we go along.

Consider the collection of numbers

$$\mathbf{S} = \{2, 2, 5, 6, 7\}.$$

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We can consider all samples from our collection of size 1. There are 5 of them and again, the mean of the samples is again 4.4. The standard deviation is 2.302.

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$$\{2, 3.5, 4, 4.5, 3.5, 4, 4.5, 5.5, 6, 6.5\}$$

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Probability	.1	.2	.2	.2	.1	.1	.1

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Our sampling distribution for samples of 3 out of our original group S is

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Theorem 6. *The sampling distribution for the mean of n measurements of some data which is known to be following a distribution of $N(\mu, \sigma)$ is itself normal, given by $N(\mu, \sigma/\sqrt{n})$.*

Example 7. *For men's heights, distributed according to*

$N(70, 4)$, *what is the probability that nine men chosen at random have an average height between* 70 *and* 73*?*

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Compare this with our previous answer as to the probability of finding a single man with such height.

Facts about sampling distributions

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Theorem 8. *Let* X *be a population with mean* μ *and standard deviation* σ*.*

Consider the sampling distribution of means of samples of size n *from our population.*

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This theorem is true no matter what the original distribution of our population is! (unlike the previous theorem)

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Before we do precisely these kinds of computations, let's see the Central Limit Theorem in action.

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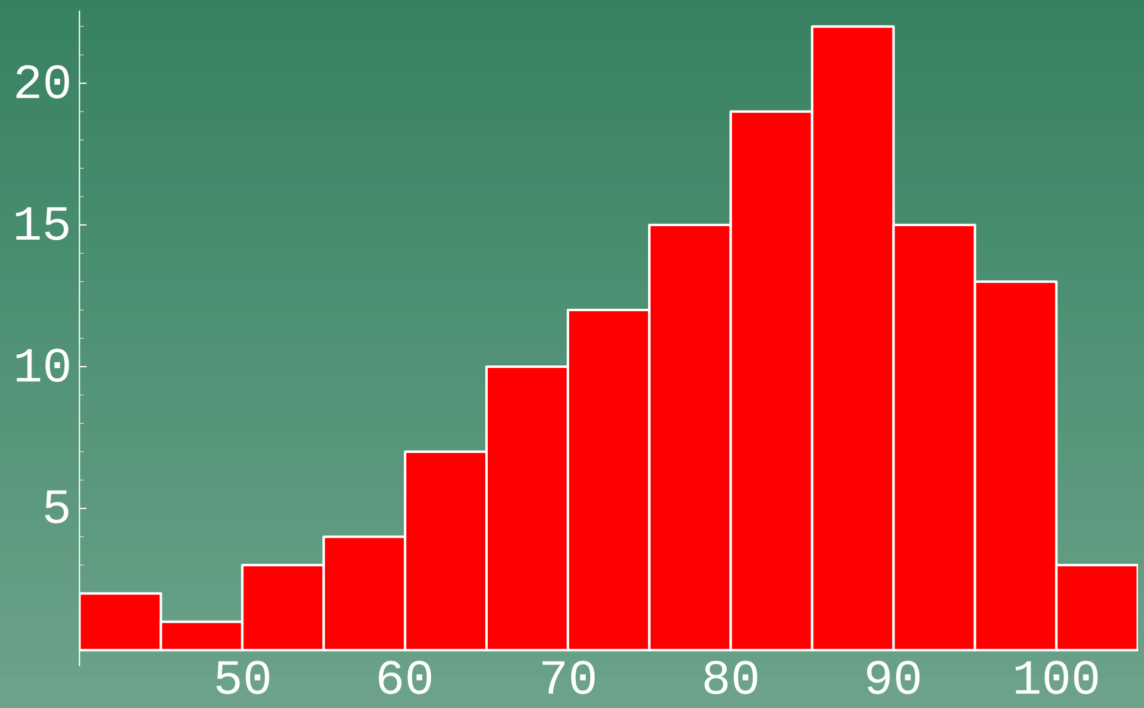

Now we take means of all samples of size 2, and look at the histogram of those numbers. (There are 7875 such samples.)

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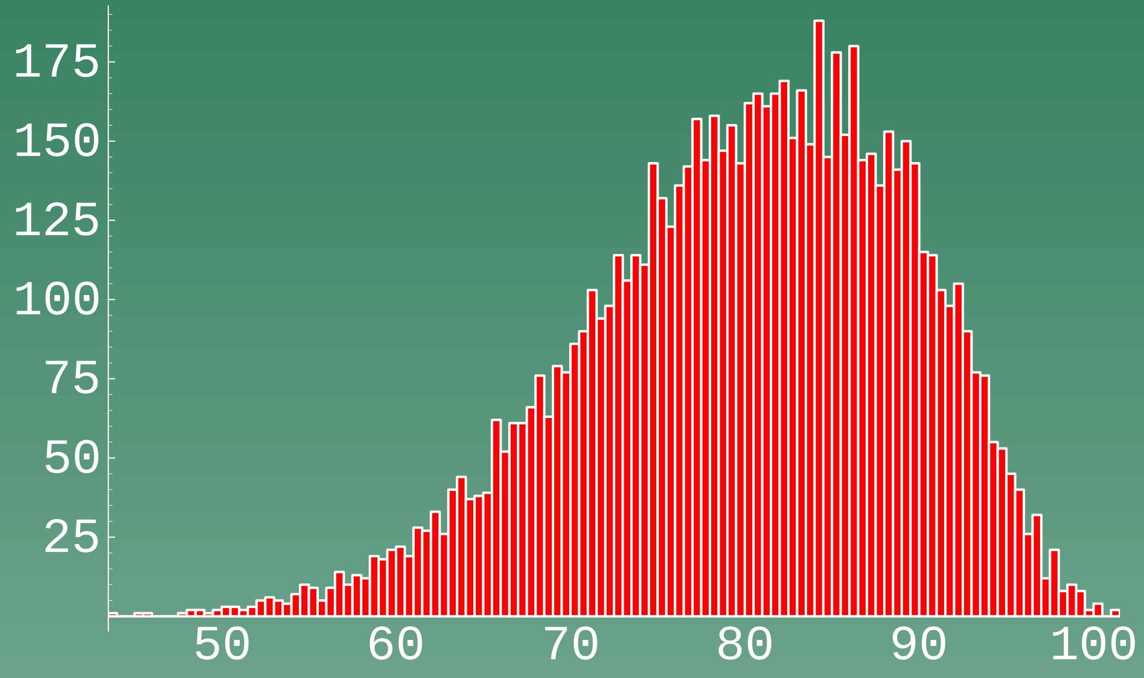
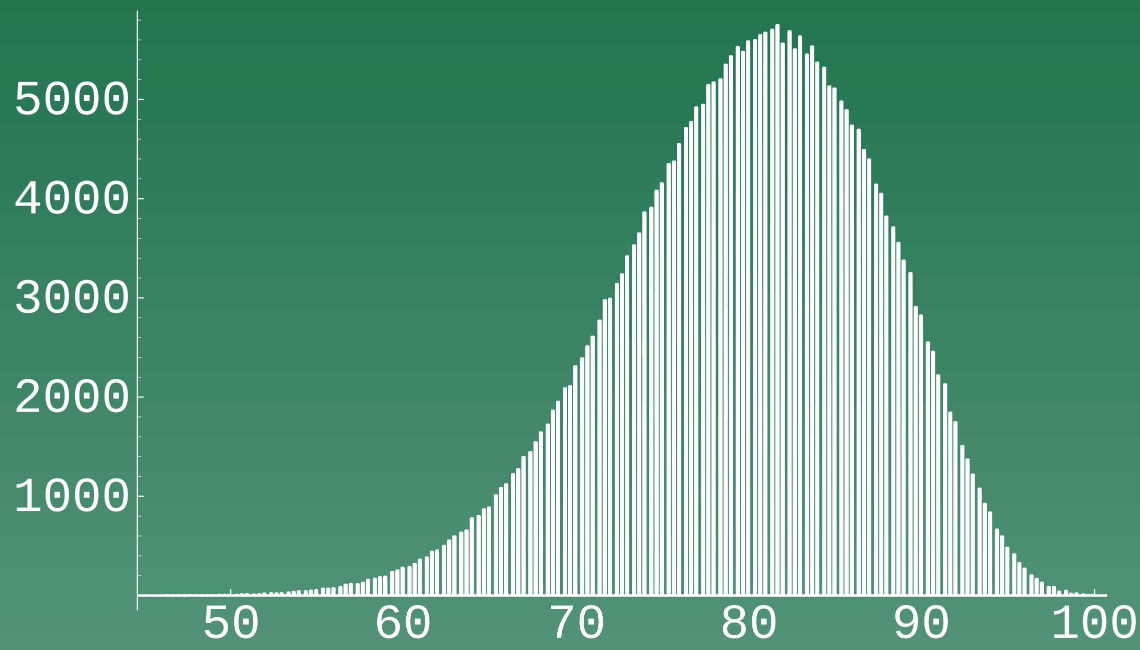


Now samples of size 3. (325500)



Example 11. *Suppose that the average price of a new car purchase is \$24145 with a standard deviation of \$3615.*

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