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Remember from last time that we are trying to understand a parameter (like the true average of purchase prices for cars in the U.S.) from a statistic (the

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Definition 1. *Given a population, the sampling distribution for samples of size* n *is the probability distribution of some parameter of the samples (for example the mean) as we go through all possible samples.*

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We go thoroughly through an example looking at sampling distributions and comment thoroughly as we go along.

Consider the collection of numbers

$$\mathbf{S} = \{2, 2, 5, 6, 7\}.$$

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We can consider all samples from our collection of size 1. There are 5 of them and again, the mean of the samples is again 4.4. The standard deviation is 2.302.

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$$\{\{2,2\},\{2,5\},\{2,6\},\{2,7\},\{2,5\},\{2,6\},$$
$$\{2,7\},\{5,6\},\{5,7\},\{6,7\}\}.$$

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Our means are

$$\{2, 3.5, 4, 4.5, 3.5, 4, 4.5, 5.5, 6, 6.5\}$$

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Probability	.1	.2	.2	.2	.1	.1	.1

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Our sampling distribution for samples of 3 out of our original group S is

X	3	3.333	3.667	4.333	4.667	5	6
Probability	.1	.1	.1	.2	.2	.2	.1

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Example 2. *Calculate the sampling distribution for the mean of samples of three out of the data set* $\{-1, 0, 0, 2, 3, 6\}$.

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Consider the sampling distribution of means of samples of size n from our population.

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This theorem is true no matter what the original distribution of our population is! (unlike the previous theorem)

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Before we do precisely these kinds of computations, let's see the Central Limit Theorem in action.

Example 5. *We look at the grade distribution for an exam.*

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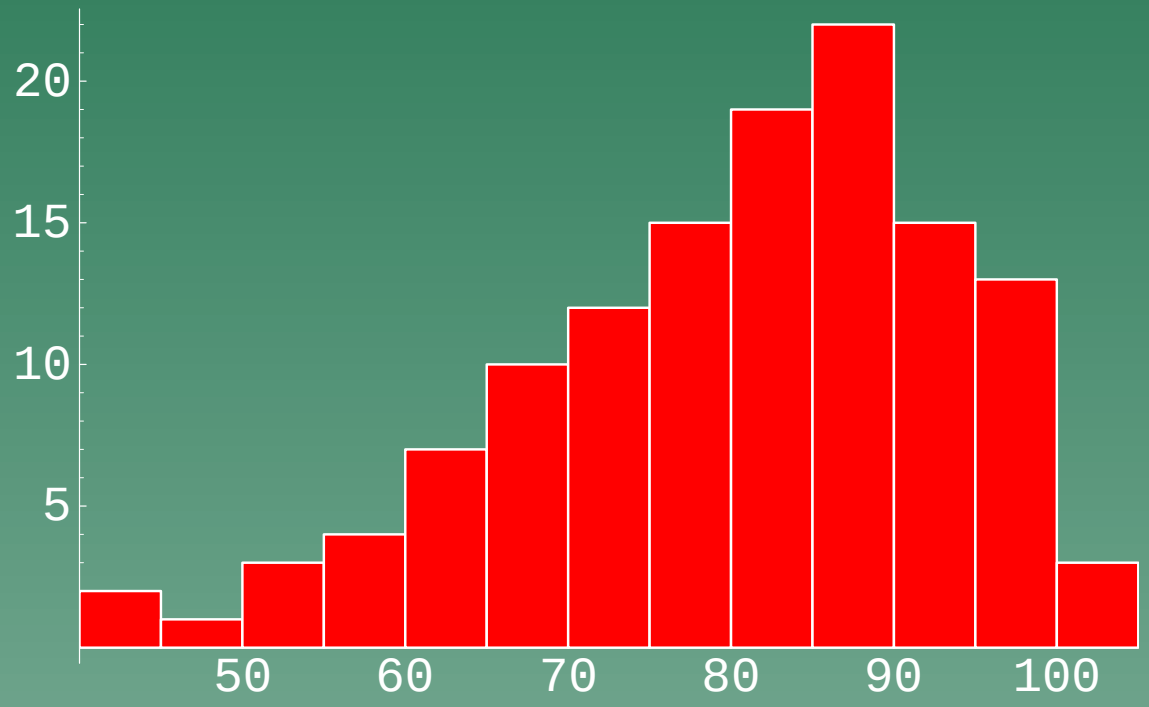

Now we take means of all samples of size 2, and look at the histogram of those numbers. (There are 7875 such samples.)

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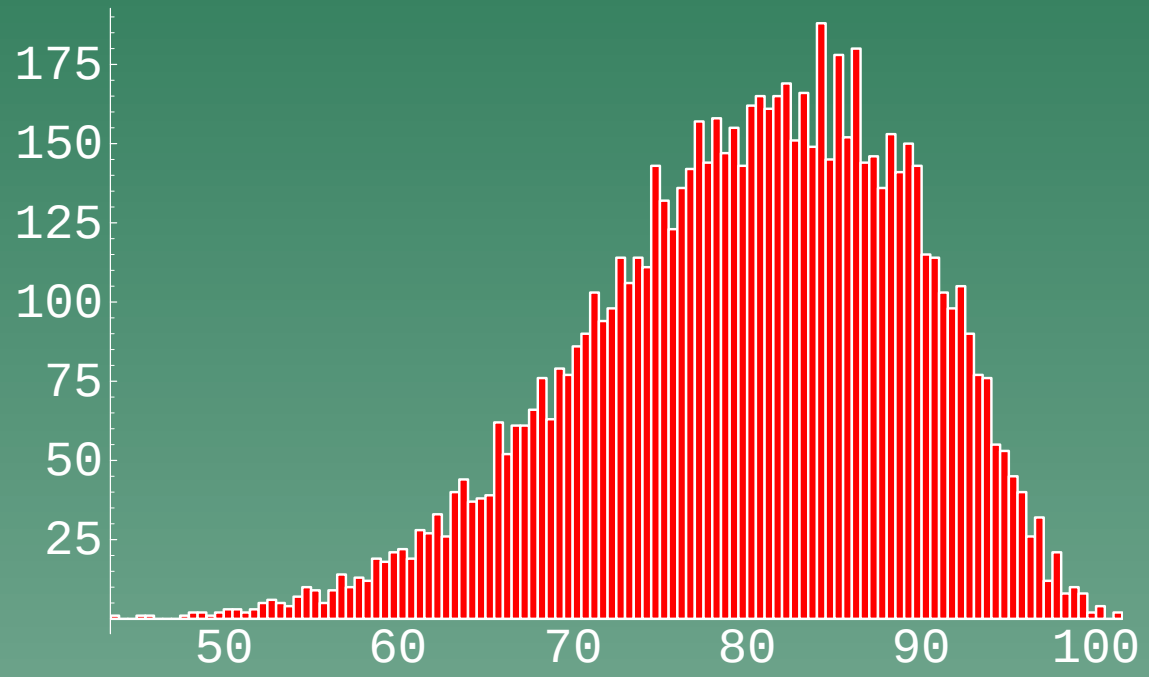
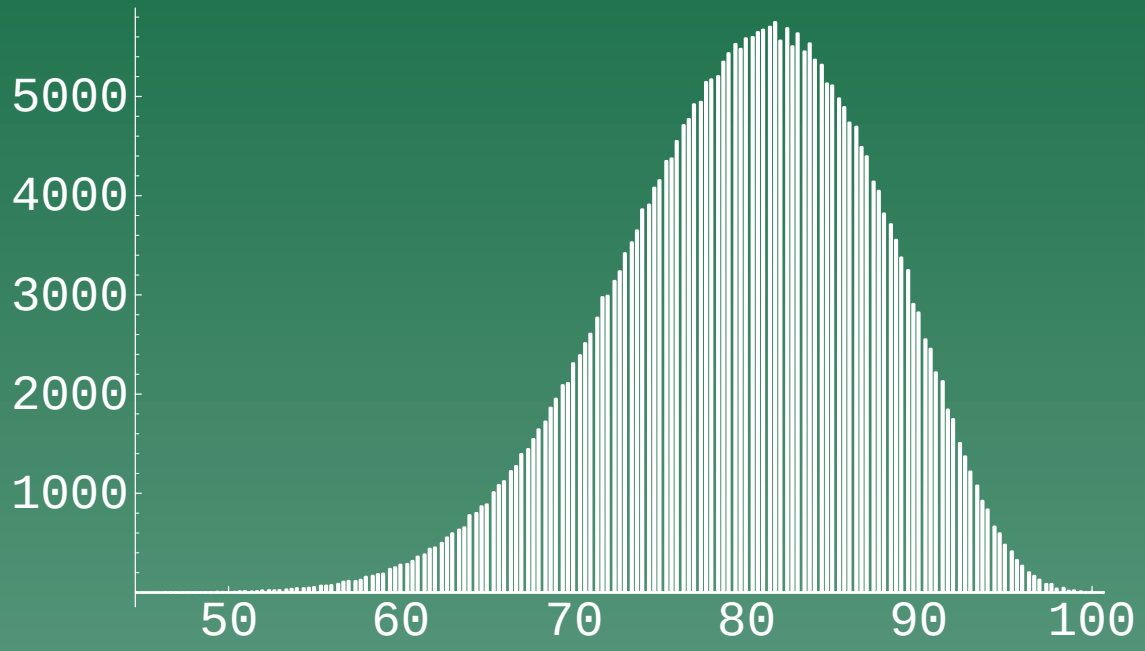


Now samples of size 3. (325500)



Example 6. *Suppose that the average price of a new car purchase is $24145 with a standard deviation of $3615.*

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Example: Process Control

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- Imagine a manufacturing process for, say, ball bearings. The bearings are supposed to be 10 mm in diameter. In fact, when the manufacturing process is working correctly, they are distributed normally $N(10, .7)$.

- We can't check every bearing as it is too time consuming. Every hour we take a sample of 10 bearings, and take the mean diameter. The sample distribution of the means, $\overline{x}$ should be $N(10, .7/\sqrt{10}) = N(10, .221)$.

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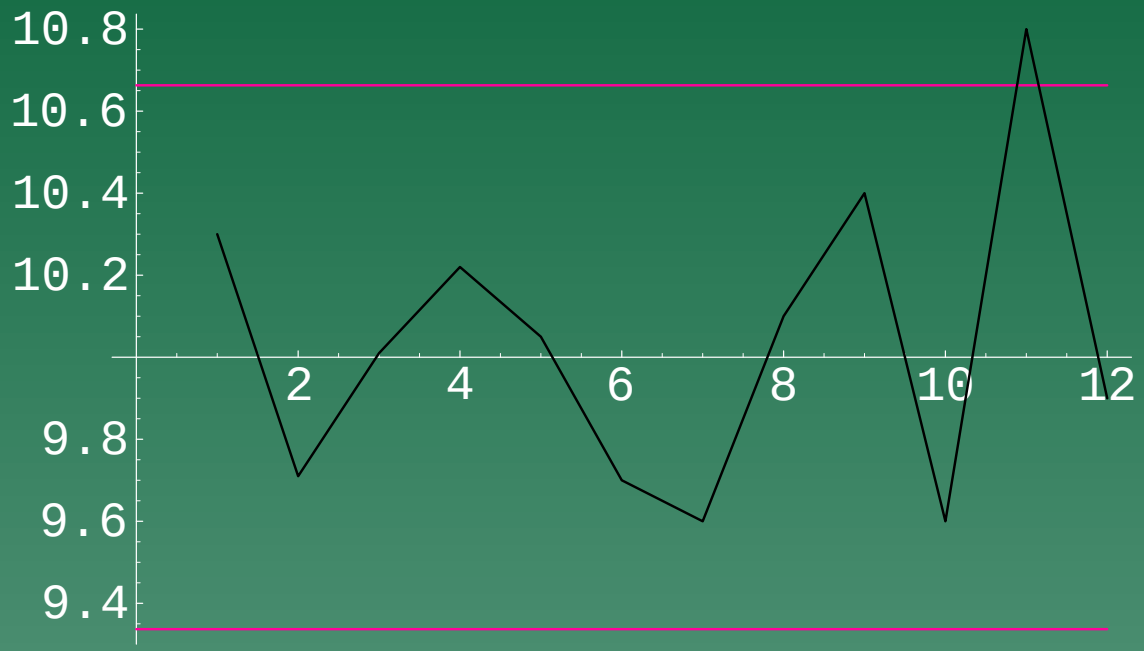
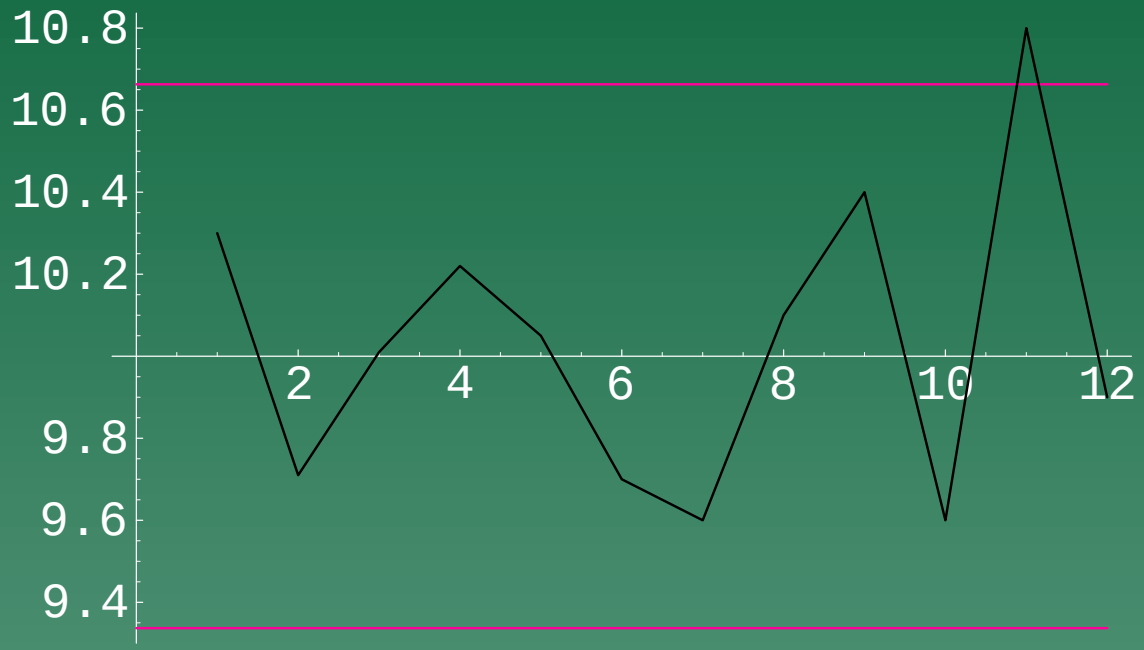

10.8
10.6
10.4
10.2
2
4
6
8
10
12
9.8
9.6
9.4



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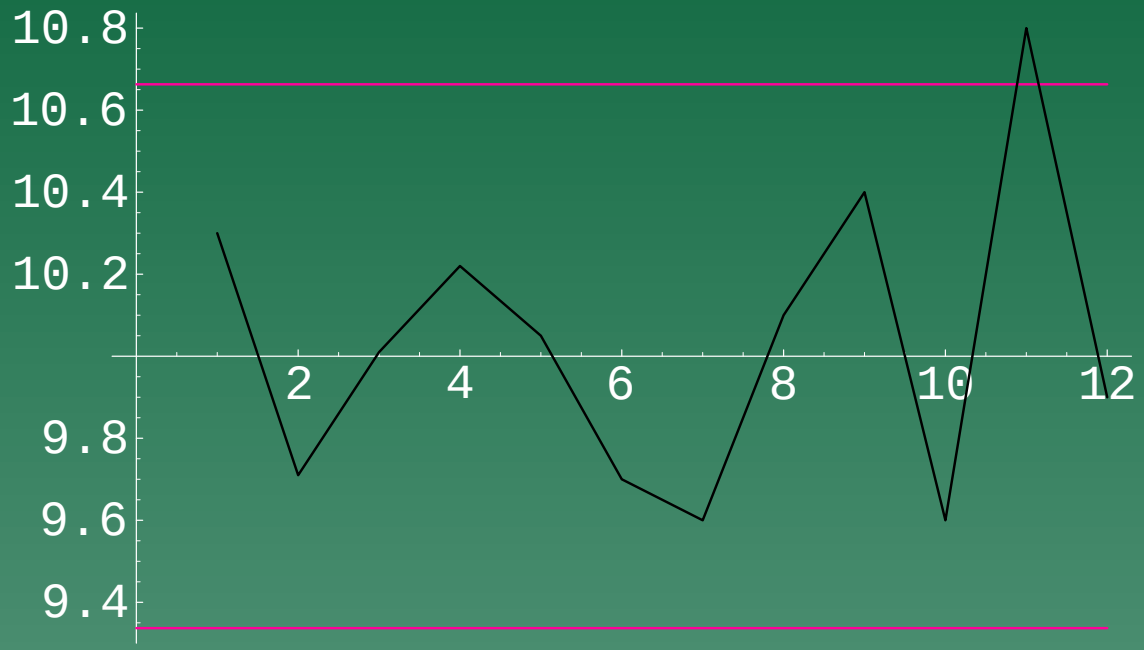

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