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In fact, these two procedures are logically related (seeing how helps us better understand both concepts).

Fact 1. *If some guess for a mean* μ_0 *is not within a* $1-\alpha$ *confidence interval about an observed mean* $\bar{x}$ *then we may reject the null hypothesis* $\mu = \mu_0$ *at significance level* α.

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Example 2. *Suppose a variable has a standard deviation of* 5 *and a measured mean of* 172.1 *from a sample size of* 80*.*

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Example 2. *Suppose a variable has a standard deviation of 5 and a measured mean of 172.1 from a sample size of 80.*

- *Find a confidence interval with* $C = 95\%$ *for this variable.*

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- *Show that the null-hypothesis of a mean equal to* 175 *can be rejected at level* 5%.

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Example 3. *What if in our UO height example, two of the sample taken were members of the basketball team over 80 inches tall?*

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Example 3. *What if in our UO height example, two of the sample taken were members of the basketball team over 80 inches tall? If these outliers are thrown out,* *what is our new confidence level for the hypothesis that UO men are above average height?*

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Estimating the mean *without* knowing σ

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In §13 the method we learned for estimating our population mean μ had the serious drawback that we had to know the standard deviation for our population. We now wish to approximate μ *without* knowing the standard deviation.

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Calculate the mean of our sample $\overline{x}$. Use s to estimate σ and then techniques we've already learned to estimate μ from $\overline{x}$.

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Recall that if our population has distribution $N(\mu, \sigma)$ and we look at samples of size n, our standardized sample mean

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$$

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The t-statistic looks much like the z-statistic we used when we assumed knowledge of the true deviation. We give the t-statistic a distinct name because unlike the z-statistic it is *not* normally distributed. If we applied the 68-95-99.7 rule, we would get incorrect answers!

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Example 4. *Use t-statistics to estimate population mean with confidence 95% if we have an SRS of size $n = 11$ with $\overline{x} = 27$ and $s = 2$.*

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Example 5. *With data as above, say whether or not the null-hypothesis of $\mu = 30$ can be rejected.*

Example 6. *If you sample* 200 *bacterial lifespans and find an average of* 10.41 *days and a deviation of* 2.1, *does this finding support the hypothesis that these bacterial lifespans are on average more than 10 days?*