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Example 1. *Find the probability of getting exactly one four when rolling a die three times.*

Example 2. *Find the probability that you there is a pair dealt in a hand with five cards.*

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- The probability distribution of a random variable is the assignment of probabilities to the values in the sample

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$$D = \begin{cases} \frac{1}{12} & 1 \leq x \leq 3 \\ \frac{1}{2} & 3 \leq x \leq 4 \\ \frac{1}{9} & 4 \leq x \leq 7 \\ 0 & \textit{otherwise} \end{cases}$$

- *What is the state space?*

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- *Which is more likely, that* X *is between* 3 *and* 4 *or that* X *is between* 5 *and* 7*?*

Compare the answer to the last question to the answer you would get if X were uniformly distributed.

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