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We will repeat our main goal many times in many ways. Here's an easy question to remember: how does one compute the "margin of error" for a poll? How does Gallup know that 65% plus or minus 4% of Americans like chocolate chip cookies? Do they really know that, anyways?

Remember from last time that we are trying to understand a parameter (like the true average of purchase prices for cars in the U.S.) from a statistic (the average of say 1000 of those purchases picked at random.

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Theorem 1. *The sampling distribution of means of random samples of size* n *from a population with mean* μ *and standard deviation* σ *is approximately*

$$N(\mu, \sigma/\sqrt{n})$$

when n *is large.*

Example 2. *Suppose that the average price of a new car purchase is \$24145 with a standard deviation of \$3615.*

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Example: Process Control

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The Central Limit Theorem has many applications, since sampling can be useful well beyond the realms of surveys and opinion polls.

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We can't check every bearing. Every hour we take a sample of 10 bearings, and take the mean diameter. The sample distribution of the means, $\overline{x}$ should be $N(10, .7/\sqrt{10}) = N(10, .221)$.

This means (by the 68-95-99.7 rule) 99.7% of the means will occur

$$10 - 3(.221) < \overline{x} < 10 + 3(.221)$$

$$9.337 < \overline{x} < 10.663$$

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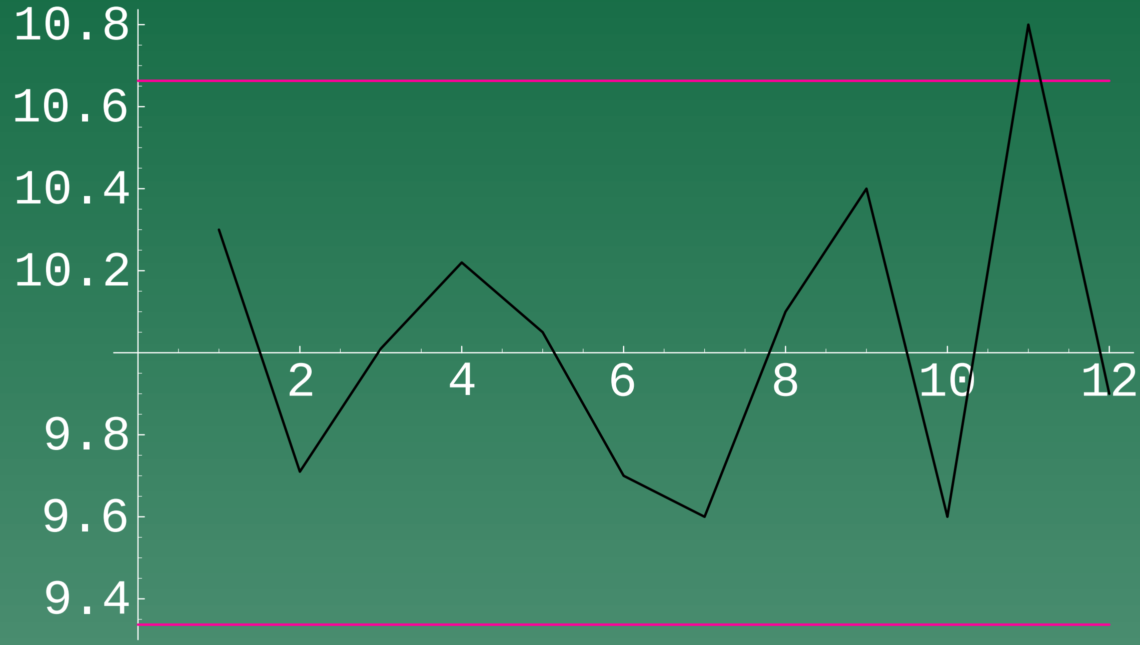
10.8
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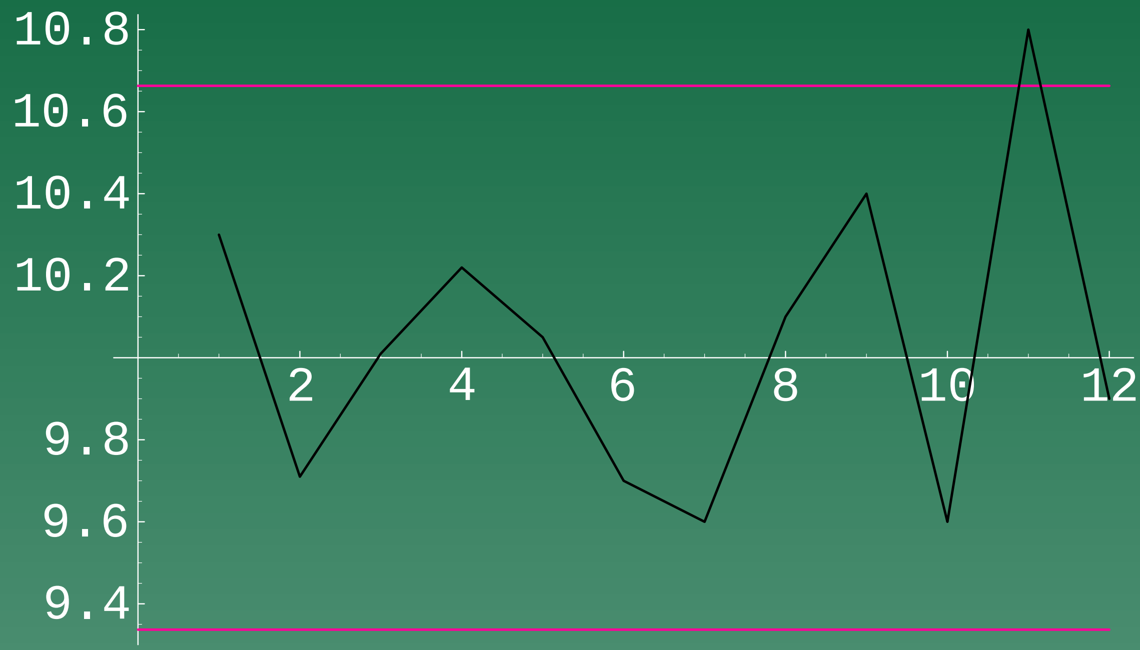

Notice the entry above the upper control line. Since 99.7% of the entries should be between the control lines, we should only get entries outside of that range about

3/1000 times.

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So an entry above the upper control line should be a rare event and should mean that we check our production line to see if problems have developed.

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Example 5. *Suppose we know that the standard deviation for new car purchases is \$3000 and we run a survey of 700 purchases which finds a mean purchase price of 23,142. Find a confidence interval for the actual mean with a 95% certainty.*

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Example 6. *We know that the standard deviation for heights of women over the entire U.S. cannot be more than* 5 *inches. Suppose that we find a random sample of 400 women which has an average height of* 63.5 *inches.*

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