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A fundamental philosophical truth is that everything changes. In physics, the change in position is known as velocity or speed. In economics, the change in price is known as inflation. In business, the change in costs is sometimes known as trend. In mathematics, the change in values of a function is known as the derivative. But to understand the derivative, which will measure "instantaneous" change, you need to to first be comfortable with "average" change over some intervals.

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Questions to ponder:

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Questions to ponder: Do these calculations necessarily mean you went 50mph for four hours?

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What do you notice about this and the previous

problem?

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Example 4. *Some functions will be sketched on the board; find the slopes of secant lines as indicated.*

Formula for average rate of change

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Definition 5. *In general, the average rate of change of some function* $f(x)$ *as* x *varies between values* a *and* b *is*

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Definition 5. *In general, the average rate of change of some function $f(x)$ as x varies between values a and b is*

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This can be computed in any way that f is presented, through a formula, through a graph, or in a table.

Example 6. *Find the average rate of change for \$1000 invested at a rate of five percent over four years.*

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Example 7. *Analyze different measured and predicted rates of change for world population according to:*
`http://www.unfpa.org/6billion/pages/worldpopgrow`

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Before formalizing it, which is difficult, we will try to understand it in examples parallel to those we have done for average rate of change.

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Example 8. *How fast was the ball falling two seconds after it was dropped from the Tower of Pisa? Five second after?*

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