MATH 81: EXPERIMENTATION AND PROOF IN MATHEMATICS

Math 81 is a course intended for students with a strong interest in mathematics and science. It is a "bridge course", a course intended to facilitate the transition from introductory to advanced classes. This is the first offering of Math 81 at Brown, so we hope you are eager to try something new.

1. Philosophy

A truism which teachers of mathematics like to repeat is that "one must learn by doing." But what does it mean to do mathematics? Judging from your high-school and introductory courses, doing mathematics could be equated with going through exercises in a book. Is this what professional mathematicians do for a living? Do math professors quiz each other over lunch? No. Doing mathematics, either applied (in academia or industry) or theoretical, is an open-ended affair. The cut-and-dried techniques one learns for solving problems (use the first derivative in order to find a maximum of a function; use modular arithmetic to encode a message) were the result of years of toil by researchers who generated examples, "played around", got stuck, shopped for relevant ideas, got inspired while on a walk, etc.

Classes in the lecture-homework-test style are probably the most efficient format in which to learn cut-and-dried techniques. How can one learn about the process of creating mathematics, of starting from scratch and building the relevant structures and techniques on your own? Here we turn to our colleagues in science departments who exhort the virtues of laboratory courses for letting students get a "first-hand" feel for science. But mathematicians do not have labs, do they? Well, not really, but we do for the most part follow the scientific method. That is, we often

- Accumulate examples of a phenomenon, that is, data.
- Examine the data and try to find patterns and relationships.
- Formulate precise conjectures which explain the patterns and relationships.
- Test conjectures by generating new data against which one may compare.

Additionally, mathematicians take a step which sets us apart from physical sciences.

- Devise an argument, known as a proof, which establishes that our conjecture always holds.

This last step in the "mathematical method", finding a proof, gives mathematics a unique character and position among intellectual disciplines and greatly shapes our methodology.

While we have no labs in which you may work, we can nonetheless create conditions under which it will be natural for you to try your hand at the mathematical method. The most difficult steps are the first and last. Generating examples of phenomena in the way mathematicians usually do requires a fair amount of experience. We are aided in this class by computers, which can generate vast amounts of concrete data. Proofs are also difficult for the novice, in any setting. This class should be a good introduction to proofs, since you will mainly be asked to prove conjectures which you have made yourself. Hence, you will often have some sense of statements you are trying to prove should be true.

We should perhaps share why one would want to master the mathematical method and where this course might lead you to go next. First, we should elaborate one sense in which mathematics is unique among scientfic pursuits, namely the use of proofs. Modulo deep philosophical difficulties pointed out by Gödel and others, the technique of proof ultimately gives mathematics an absolute certainty not shared by any other field. While Newton's laws of gravitation were modified by Einstein, and Einstein's will one day be modified, the Pythagorean theorem is as true today as it has always been and could never be made more true. In order to achieve this kind of certainty, mathematics demands of its practicioners a particular mental discipline and creativity. This fostering of mental discipline and creativity is a strong argument for studying mathematics as part of a liberal education.

For purposes of sharing their work, however, mathematicians perhaps put too much of an emphasis on proof, and not enough on techniques of discovery. Indeed if asked casually, most mathematicians would abbreviate their description of the mathematical method to mention only conjectures and proofs. Moreover, the bulk of mathematical writing is devoted to precise statements of theorems and proofs. Unlike the situation physical sciences, the techniques one uses for making discoveries are rarely documented. This emphasis on precise statements and proof can give mathematics a rarified air. From the outside, one does not see mathematics as a living subject.

The ultimate philosophical goal of this class is to introduce you to mathematics as a living, growing subject. We hope this will motivate you to take more advance mathematics courses, and give you more perspective on those courses when you do. We also feel that much of what we do will be relevant to other pursuits, in particular in the sciences. Indeed, getting to know mathematical discovery and proof can be an integral part of a liberal education.

2. Details

This course meets Tuesday and Thursday at 1 pm in room 265 of the CIT. During most class sessions we will have a bit of time at the beginning and/ or end of class to have discussions in which either I introduce material you are about to explore or you share results you have found, which we discuss as a class. The rest of class time will be devoted to your working on a project in teams of two. The TA, Zack Mesyan, and I will go around the room and ask questions, give suggestions, etc. to help you along in your explorations. Most of these explorations will be made with the help of the computer. In particular, we will be running BASIC programs which were written for use with our textbook, "Laboratories in Mathematical Experimentation." We hope you eventually become comfortable with modifying these programs to take the exploration in directions not suggested in the book.

Your grade will depend almost entirely on your write-ups of your work, as well as a presentation at the end of the term (80% write-ups, 10% presentation, 10% class participation). We will be more specific about what the write-ups are to entail when the first one is due. Suffice it to say that you will write up enough background to make your work understandable to a peer who is not in the class, important pieces of data you generated, and conjectures and proofs which resulted from your analysis of the data. We will look very kindly on explorations and write-ups which go beyond what is covered in the text. Write-ups will be due the first Tuesday after a section is completed. You will be given a chance for revision, and your final grade on the write-up will be the average of the two papers you turn in (or just the first one if you choose not to revise). It is strongly suggested that you take advantage of the TA as a resource for comments on drafts before you turn in the first version.

In the first day of class we will do some exploration as a class which does not require a computer. On the second day, we will go through the highlights of Chapter One on Iteration of Linear Functions. In this way, we will learn some basics of computer exploration. The class starts in earnest on the third day, September 12th, on which we will delve into the chapter on Graph Coloring. Typically, we will spend three class sessions on a chapter. Other chapters we plan to cover are: Randomized Response Surveys, Polyhedra, Sequences and Series, p-adic Numbers, and Iteration of Quadratic Functions. There will be a final project whose topic you will find on your own (through sources such as "Mathematics Magazine"), on which you will make a final presentation, in addition to handing in a write-up, at the end of the term. We expect to devote three class sessions to each chapter (except for the first). As this is the first time this class is being offered, we reserve the right to be a bit flexible with the syllabus and go through chapters either more or less quickly, covering more or fewer chapters in the end. Because this is a new class, feel encouraged to give feedback on any aspect of the course.

3. Instructor and TA Information

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