Learning and Expectations in Macroeconomics
Problems for Chapter 8

1. Consider the basic linear univariate model of Chapter 8, Section 2

$$y_t = \alpha + \beta_0 E^*_{t-1} y_t + \beta_1 E^*_{t-1} y_{t+1} + v_t.$$

(i) Using the method of undetermined coefficients, i.e. trying solutions of the form

$$y_t = a + b y_{t-1} + v_t + c v_{t-1} + d \varepsilon_{t-1},$$

where ε_t is an arbitrary martingale difference sequence, show that equations (8.3) and (8.4) are obtained under rational expectations.
(ii) Introduce an additional lag on y_t in the guessed form, i.e.

$$y_t = a + b y_{t-1} + e y_{t-2} + v_t + c v_{t-1} + d \varepsilon_{t-1},$$

Show that in an REE $e = 0$, that is, including higher order lags does not enlarge the set of solutions.

2. Consider the overlapping wage-contract model (Example 3 of Chapter 8, Section 5). Show that the model is saddle-point stable and that the stationary MSV solution is E-stable.

3. Consider the Cagan model with policy feedback (see Example 8 of Chapter 8, Section 6).
(i) Show that the model has two $AR(1)$ solutions, provided the parameters satisfy appropriate restrictions. Derive the E-stability conditions for an $AR(1)$ solution under the assumption that the time t information set includes p_t as well as the exogenous variables dated t.
(ii) Assume the numerical value $\gamma = 0.5$ and consider two alternative values for the policy feedback parameter $d = 0.2$ or 0.9. Is there an E-stable solution for each of these two cases?

4. Consider the univariate model

$$y_t = \alpha + \delta y_{t-1} + \beta E^*_{t-1} y_{t+1} + v_t,$$

where v_t is white noise.

(i) Consider MSV (minimal state variable) rational expectations solutions of the AR(1) form

$$y_t = a + b y_{t-1} + v_t.$$

Assuming an appropriate condition is met, show that there exist (generically) two such solutions and solve for the RE values of a and b.

(ii) What are the conditions for E-stability of an AR(1) solution? Can both solutions be E-stable?

(iii) Anticipating Chapter 9, the E-stability conditions computed in (ii) are called "weak E-stability" conditions. Suppose that the perceived law of motion is overparameterized as an AR(2) process, i.e. as

$$y_t = a + by_{t-1} + cy_{t-2} + v_t.$$

If $(\bar{a}, \bar{b})$ is an RE AR(1) solution, this can be viewed as an AR(2) solution with $c = 0$. Compute the T-map from the PLM (a, b, c) to the ALM $T(a, b, c)$ and corresponding E-stability conditions for the AR(1) solutions. These conditions are called "strong E-stability conditions." In this model, is a weakly E-stable AR(1) solution necessarily strongly E-stable?

5. Suppose

$$\begin{aligned} y_t &= \beta E_t^* y_{t+1} + w_t, \\ w_t &= \rho_1 w_{t-1} + \rho_2 w_{t-2} + v_t, \end{aligned}$$

where v_t is white noise.

(i) Find the MSV rational expectations solution of the form

$$y_t = aw_t + bw_{t-1}.$$

(ii) Obtain the E-stability conditions for this solution. (Note: the roots of a 2×2 matrix have negative real parts if its trace is negative and determinant is positive). Show these conditions are satisfied if $|\beta|$ is small enough.

6. Consider the model

$$y_t = \alpha + \beta_0 E_{t-1}^* y_t + \beta_1 E_{t-1}^* y_{t+1} + \beta_2 E_{t-1}^* y_{t+2} + v_t + \kappa w_{t-1},$$

where v_t is a white noise disturbance and w_t is an observable *iid* random variable with mean $E(w_t) = \mu$.

(i) Using the method of undetermined coefficients, find the MSV (minimum state variable) rational expectations solution of the form

$$y_t = a + bw_{t-1} + v_t.$$

That is, find the values $(\bar{a}, \bar{b})$ which generate an REE.

(ii) For the PLM (perceived law of motion) of the form given in (i) give the map $T(a, b)$ from the PLM to the ALM (actual law of motion) and obtain the corresponding E-stability conditions.

(iii) Consider now real time learning in which agents estimate the coefficients of their forecast function. Suppose that agents estimate the model $y_t = a + bw_{t-1} + v_t$ using least squares. Write down the equations that define the recursive dynamic system under least squares learning. Under what conditions does the system converge to the MSV solution?

7. Consider the model

$$y_t = \alpha + \beta E^*_{t-1} y_{t+1} + v_t,$$

where v_t is white noise. (You should assume $\beta \neq 1$).

(i) Find the MSV solution, which takes the form $y_t = a + v_t$. Use the method of undetermined coefficients to show that there is also an RE solution of the form

$$y_t = a + b y_{t-1} + v_t,$$

where $b \neq 0$.

(ii) For PLMs of the form $y_t = a + b y_{t-1} + v_t$, give the map from the PLM to the ALM and obtain the corresponding E-stability conditions for an RE solution $(\bar{a}, \bar{b})$. What do these imply about the E-stability of the MSV solution and of the solution $y_t = a + b y_{t-1} + v_t$ with $b \neq 0$