HOMEWORK ASSIGNMENT #2 MATH 415-515 SPRING 2008

Do 2.12, 2.13., 2.21, 2.22, 2.23, 2.24, 2-25, 2-26. Also do:

Problem 2.A Let V be the vector space of all $n \times n$ real matrices – note that $\dim(V) = n^2$. Give V the operatornorm – it really doesn't matter – recall that ALL norms on a finite dimensional real vector space are comparable. Let

$$f(x) := \sum_{n=0}^{\infty} \frac{x^n}{n!} .$$

(1) Show that this series converges for any $x \in V$. (Hint: $||xy|| \leq ||x|| \cdot ||y||$).
(2) Show that f is differentiable at the origin and that $f'(0)\xi = \xi$ for any $\xi \in V$. Thus $f'(0) = \mathrm{id}$.
(3) (Worth 5 bonus points). Show that f is differentiable at any point $x \in V$. Ekaterina thinks that

$$\begin{aligned} f'(x)\xi &= \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \{yx^{n-1} + xyx^{n-2} + ... + x^{n-2}yx + x^{n-1}y\} . \end{aligned}$$

Problem 2.B Let $\mathcal{P}_n$ denote the set of all polynomials of degree n. This is a real vector space of real dimension $n+1$. Define $f(p) := \cos(p'(0))$. This is a map from $\mathcal{P}_n$ to $\mathbb{R}$. Show that f is differentiable at any $p \in \mathcal{P}_n$ and determine $f'(p)q$.