

EXERCISES

4.1. For each set below that is bounded above, list three upper bounds for the set. Otherwise write "NOT BOUNDED ABOVE" or "NBA."

- |  |   |
|--|---|
| (a) $[0, 1]$   | (b) $(0, 1)$                                |
| (c) $\{2, 7\}$   | (d) $\{\pi, e\}$                            |
| (e) $\{1/n : n \in \mathbb{N}\}$                                 | (f) $\{0\}$                                 |
| (g) $[0, 1] \cup [2, 3]$   | (h) $\bigcup_{n=1}^{\infty} [2n, 2n+1]$     |
| (i) $\bigcap_{n=1}^{\infty} [-1/n, 1+1/n]$                       | (j) $\{1-1/3^n : n \in \mathbb{N}\}$        |
| (k) $\{n+(-1)^n/n : n \in \mathbb{N}\}$                          | (l) $\{r \in \mathbb{Q} : r < 2\}$          |
| (m) $\{r \in \mathbb{Q} : r^2 < 4\}$                             | (n) $\{r \in \mathbb{Q} : r^2 < 2\}$        |
| (o) $\{x \in \mathbb{R} : x < 0\}$                               | (p) $\{1, \pi/3, \pi^2, 10\}$               |
| (q) $\{0, 1, 2, 4, 8, 16\}$                                      | (r) $\bigcap_{n=1}^{\infty} (1-1/n, 1+1/n)$ |
| (s) $\{1/n : n \in \mathbb{N} \text{ and } n \text{ is prime}\}$ | (t) $\{x \in \mathbb{R} : x^3 < 8\}$        |
| (u) $\{x^2 : x \in \mathbb{R}\}$                                 | (v) $\{\cos(n\pi/3) : n \in \mathbb{N}\}$   |
| (w) $\{\sin(n\pi/3) : n \in \mathbb{N}\}$                        |   |

4.2. Repeat Exercise 4.1 for lower bounds.

4.3. For each set in Exercise 4.1, give its supremum if it has one. Otherwise write "NO sup."

4.7. Let  $S$  and  $T$  be nonempty bounded subsets of  $\mathbb{R}$ .

- (a) Prove that if  $S \subseteq T$ , then  $\inf T \leq \inf S \leq \sup S \leq \sup T$ .  
 (b) Prove that  $\sup(S \cup T) = \max\{\sup S, \sup T\}$ . Note: In part (b) do not assume  $S \subseteq T$ .

4.14. Let  $A$  and  $B$  be nonempty bounded subsets of  $\mathbb{R}$  and let  $S$  be the set of all sums  $a+b$  where  $a \in A$  and  $b \in B$ .

- (a) Prove that  $\sup S = \sup A + \sup B$ .  
 (b) Prove that  $\inf S = \inf A + \inf B$ .

EXERCISES

5.1. Write the following sets in interval notation:

- |                                    |   |
|------------------------------------|---|
| (a) $\{x \in \mathbb{R} : x < 0\}$ | (b) $\{x \in \mathbb{R} : x^3 \leq 8\}$ |
| (c) $\{x^2 : x \in \mathbb{R}\}$   | (d) $\{x \in \mathbb{R} : x^2 < 8\}$    |

5.2. Give the infimum and supremum of each set listed in Exercise 5.1.

5.3. Give the infimum and supremum of each unbounded set listed in Exercise 4.1.

5.4. Let  $S$  be a nonempty subset of  $\mathbb{R}$  and let  $-S = \{-s : s \in S\}$ . Prove that  $\inf S = -\sup(-S)$ . Hint: For the case  $-\infty < \inf S$ , simply state that this was proved in Exercise 4.9.

5.5. Prove that  $\inf S \leq \sup S$  for every nonempty subset of  $\mathbb{R}$ . Compare Exercise 4.6(a).

5.6. Let  $S$  and  $T$  be nonempty subsets of  $\mathbb{R}$  such that  $S \subseteq T$ . Prove that  $\inf T \leq \inf S \leq \sup S \leq \sup T$ . Compare Exercise 4.7(a).