## Exercises

7.1. Write out the first five terms of the following sequences.

(a) $s_n = 1/(3n+1)$ (b) $b_n = (3n+1)/(4n-1)$
(c) $c_n = n/3^n$ (d) $\sin(n\pi/4)$

7.2. For each sequence in Exercise 7.1, determine whether it converges. If it converges, give its limit. No proofs are required.

7.3. For each sequence below determine whether it converges and, if it converges, give its limit. No proofs are required.

(a) $a_n = n/(n+1)$ (b) $b_n = (n^2+3)/(n^2-3)$
(c) $c_n = 2^{-n}$ (d) $t_n = 1 + 2/n$
(e) $x_n = 73 + (-1)^n$ (f) $s_n = (2)^{1/n}$
(g) $y_n = n!$ (h) $d_n = (-1)^n n$
(i) $(-1)^n/n$ (j) $(7n^3+8n)/(2n^3-31)$
(k) $(9n^2-18)/(6n+18)$ (l) $\sin(n\pi/2)$
(m) $\sin(n\pi)$ (n) $\sin(2n\pi/3)$
(o) $(1/n)\sin n$ (p) $(2^{n+1}+5)/(2^n-7)$
(q) $3^n/n!$ (r) $(1+1/n)^2$
(s) $(4n^2+3)/(3n^2-2)$ (t) $(6n+4)/(9n^2+7)$

7.4. Give examples of

(a) a sequence $(x_n)$ of irrational numbers having a limit $\lim x_n$ that is a rational number,
(b) a sequence $(r_n)$ of rational numbers having a limit $\lim r_n$ that is an irrational number.

7.5. Determine the following limits. No proofs are required, but show any relevant algebra.

(a) $\lim s_n$ where $s_n = \sqrt{n^2+1} - n$,
(b) $\lim(\sqrt{n^2+n} - n)$,
(c) $\lim(\sqrt{4n^2+n} - 2n)$.

*Hint for* (a): First show that $s_n = 1/(\sqrt{n^2+1} + n)$.

## Exercises

Formal proofs are required in the following exercises.

8.1. Prove the following:

(a) $\lim[(-1)^n/n] = 0$ (b) $\lim(1/n^{1/3}) = 0$
(c) $\lim[(2n-1)/(3n+2)] = 2/3$ (d) $\lim(n+6)/(n^2-6) = 0$

8.2. Determine the limits of the following sequences and then prove your claims.

(a) $a_n = n/(n^2+1)$ (b) $b_n = (7n-19)/(3n+7)$
(c) $c_n = (4n+3)/(7n-5)$ (d) $d_n = (2n+4)/(5n+2)$
(e) $s_n = (1/n)\sin n$

8.7. Show that the following sequences do not converge.

(a) $\cos(n\pi/3)$ (b) $s_n = (-1)^n n$
(c) $\sin(n\pi/3)$

8.8. Prove the following [see Exercise 7.5]:

(a) $\lim[\sqrt{n^2+1} - n] = 0$
(b) $\lim[\sqrt{n^2+n} - n] = 1/2$
(c) $\lim[\sqrt{4n^2+n} - 2n] = 1/4$

9.1. Using the limit theorems 9.2–9.6 and 9.7, prove the following. Justify all steps.

(a) $\lim[(n+1)/n]=1$ (b) $\lim[(3n+7)/(6n-5)]=1/2$

(c) $\lim[(17n^5+73n^4-18n^2+3)/(23n^5+13n^3)]=17/23$

9.2. Suppose that $\lim x_n=3$, $\lim y_n=7$ and that all $y_n$ are nonzero. Determine the following limits:

(a) $\lim(x_n+y_n)$ (b) $\lim[(3y_n-x_n)/y_n^2]$

9.3. Suppose that $\lim a_n=a$, $\lim b_n=b$, and that $s_n=(a_n^3+4a_n)/(b_n^2+1)$. Prove $\lim s_n=(a^3+4a)/(b^2+1)$ carefully, using the limit theorems.

9.4. Let $s_1=1$ and for $n \geqslant 1$ let $s_{n+1}=\sqrt{s_n+1}$.

(a) List the first four terms of $(s_n)$.

(b) It turns out that $(s_n)$ converges. Assume this fact and prove that the limit is $(1+\sqrt{5})/2$.

9.5. Let $t_1=1$ and $t_{n+1}=(t_n^2+2)/2t_n$ for $n \geqslant 1$. Assume that $(t_n)$ converges and find the limit.

9.6. Let $x_1=1$ and $x_{n+1}=3x_n^2$ for $n \geqslant 1$.

(a) Show that if $a=\lim x_n$, then $a=1/3$ or $a=0$.

(b) Does $\lim x_n$ exist? Explain.

(c) Discuss the apparent contradiction between parts (a) and (b).



9.8. Give the following when they exist. Otherwise assert "NOT EXIST."

(a) $\lim n^3$ (b) $\lim(-n^3)$

(c) $\lim(-n)^n$ (d) $\lim(1.01)^n$

(e) $\lim n^n$