10.1. Which of the following sequences are nondecreasing? nonincreasing? bounded?

(a) $1/n$ (b) $(-1)^n/n^2$
(c) n^5 (d) $\sin(n\pi/7)$
(e) $(-2)^n$ (f) $n/3^n$

10.2. Prove Theorem 10.2 for bounded nonincreasing sequences.

10.3. For a decimal expansion $k.d_1d_2d_3d_4\cdots$, let (s_n) be defined as in 10.3. Prove that $s_n < k+1$ for all $n \in \mathbb{N}$. *Hint*: $9/10+9/10^2+\cdots+9/10^n = 1-1/10^n$ for all n.

10.4. Discuss why Theorems 10.2 and 10.11 would fail if we restricted our world of numbers to the set $\mathbb{Q}$ of rational numbers.

10.5. Prove Theorem 10.4(ii).

10.6. (a) Let (s_n) be a sequence such that

$$|s_{n+1}-s_n| < 2^{-n} \qquad \text{for all} \quad n \in \mathbb{N}.$$

Prove that (s_n) is a Cauchy sequence and hence a convergent sequence.

(b) Is the result in (a) true if we only assume that $|s_{n+1}-s_n| < 1/n$ for all $n \in \mathbb{N}$?

10.7. Let S be a bounded nonempty subset of $\mathbb{R}$ and suppose $\sup S \notin S$. Prove that there is a nondecreasing sequence (s_n) of points in S such that $\lim s_n = \sup S$.

10.8. Let (s_n) be a nondecreasing sequence of positive numbers and define $(s_1+s_2+\cdots+s_n)/n$. Prove that (σ_n) is a nondecreasing sequence.

10.9. Let $s_1 = 1$ and $s_{n+1} = (n/(n+1))s_n^2$ for $n \geqslant 1$.

(a) Find s_2, s_3 and s_4.
(b) Show that $\lim s_n$ exists.
(c) Prove that $\lim s_n = 0$.

10.10. Let $s_1 = 1$ and $s_{n+1} = (s_n+1)/3$ for $n \geqslant 1$.

(a) Find s_2, s_3 and s_4.
(b) Use induction to show that $s_n > \frac{1}{2}$ for all n.
(c) Show that (s_n) is a nonincreasing sequence.
(d) Show that $\lim s_n$ exists and find $\lim s_n$.