16.1. (a) Show that $2.749\bar{9}$ and $2.750\bar{0}$ are both decimal expansions for $11/4$.
(b) Which of these expansions arises from the long division process described in 16.1?

16.4. Write the following repeating decimals as rationals, i.e., as fractions of integers.

(a) $.\bar{2}$
(b) $.0\bar{2}$
(c) $.\overline{02}$
(d) $3.\overline{14}$
(e) $.\overline{10}$
(f) $.1\overline{492}$

16.5. Find the decimal expansions of the following rational numbers.

(a) $1/8$
(b) $1/16$
(c) $2/3$
(d) $7/9$
(e) $6/11$
(f) $22/7$

16.6. Find the decimal expansions of $1/7$, $2/7$, $3/7$, $4/7$, $5/7$ and $6/7$. Note the interesting pattern.

16.7. Is $.12345678910111213141516171819202122232425 26\cdots$ rational?

17.1. Let $f(x)=\sqrt{4-x}$ for $x\leqslant 4$ and $g(x)=x^2$ for all $x\in\mathbb{R}$.

(a) Give the domains of $f+g$, fg, $f\circ g$ and $g\circ f$.
(b) Find the values $f\circ g(0)$, $g\circ f(0)$, $f\circ g(1)$, $g\circ f(1)$, $f\circ g(2)$ and $g\circ f(2)$.
(c) Are the function $f\circ g$ and $g\circ f$ equal?
(d) Are $f\circ g(3)$ and $g\circ f(3)$ meaningful?

17.2. Let $f(x)=4$ for $x\geqslant 0$, $f(x)=0$ for $x<0$, and $g(x)=x^2$ for all x. Thus $\operatorname{dom}(f)=\operatorname{dom}(g)=\mathbb{R}$.

(a) Determine the following functions: $f+g$, fg, $f\circ g$, $g\circ f$. Be sure to specify their domains.
(b) Which of the functions f, g, $f+g$, fg, $f\circ g$, $g\circ f$ is continuous?

17.3. Accept on faith that the following familiar functions are continuous on their domains: $\sin x$, $\cos x$, e^x, 2^x, $\log_e x$ for $x>0$, x^p for $x>0$ [p any real number]. Use these facts and theorems in this section to prove that the following functions are also continuous.

(a) $\log_e(1+\cos^4 x)$
(b) $[\sin^2 x+\cos^6 x]^{\pi}$
(c) 2^{x^2}
(d) 8^x
(e) $\tan x$ for $x\neq$ odd multiple of $\pi/2$
(f) $x\sin(1/x)$ for $x\neq 0$
(g) $x^2\sin(1/x)$ for $x\neq 0$
(h) $(1/x)\sin(1/x^2)$ for $x\neq 0$

17.4. Prove that the function $\sqrt{x}$ is continuous on its domain $[0,\infty)$. *Hint*: Apply Example 5 in §8.

17.5. (a) Prove that if $m\in\mathbb{N}$, then the function $f(x)=x^m$ is continuous on $\mathbb{R}$.
(b) Prove that every *polynomial function* $p(x)=a_0+a_1x+\cdots+a_nx^n$ is continuous on $\mathbb{R}$.

17.6. A *rational function* is a function f of the form p/q where p and q are polynomial functions. The domain of f is $\{x\in\mathbb{R}: q(x)\neq 0\}$. Prove that every rational function is continuous. *Hint*: Use Exercise 17.5.

17.10. Prove that the following functions are discontinuous at the indicated points. You may use either Definition 17.1 or the ϵ-δ property in Theorem 17.2.

(a) $f(x)=1$ for $x>0$ and $f(x)=0$ for $x\leqslant 0$, $x_0=0$;
(b) $g(x)=\sin(1/x)$ for $x\neq 0$ and $g(0)=0$, $x_0=0$;
(c) $\operatorname{sgn}(x)=-1$ for $x<0$, $\operatorname{sgn}(x)=1$ for $x>0$, and $\operatorname{sgn}(0)=0$, $x_0=0$;
(d) $P(x)=15$ for $0\leqslant x<1$ and $P(x)=15+13n$ for $n\leqslant x<n+1$, x_0 a positive integer.

The function sgn is called the *signum function*; note that $\operatorname{sgn}(x)=x/|x|$ for $x\neq 0$. The definition of P, the postage-stamp function *circa* 1979, means P takes the value 15 on the interval $[0,1)$, the value 28 on the interval $[1,2)$, the value 41 on the interval $[2,3)$, etc.

17.14. For each rational number x, write x as p/q where p,q are integers with no common factors and $q>0$, and then define $f(x)=1/q$. Also define $f(x)=0$ for all $x\in\mathbb{R}\setminus\mathbb{Q}$. Thus $f(x)=1$ for each integer, $f(\frac{1}{2})=f(-\frac{1}{2})=f(\frac{3}{2})=\cdots=\frac{1}{2}$, etc. Show that f is continuous at each point of $\mathbb{R}\setminus\mathbb{Q}$ and discontinuous at each point of $\mathbb{Q}$.