EXERCISES

18.1. Let f be as in Theorem 18.1. Show that if $-f$ assumes its maximum at $x_0 \in [a, b]$, then f assumes its minimum at x_0.

18.2. Reread the proof of Theorem 18.1 with $[a, b]$ replaced by (a, b). Where does it break down? Discuss.

18.3. Use calculus to find the maximum and minimum of $f(x) = x^3 - 6x^2 + 9x + 1$ on $[0, 5)$.

18.4. Let $S \subseteq \mathbb{R}$ and suppose there exists a sequence (x_n) in S that converges to a number $x_0 \notin S$. Show that there exists an unbounded continuous function on S.

18.5. (a) Let f and g be continuous functions on $[a, b]$ such that $f(a) \geqslant g(a)$ and $f(b) \leqslant g(b)$. Prove that $f(x_0) = g(x_0)$ for at least one x_0 in $[a, b]$.
(b) Show that Example 1 can be viewed as a special case of part (a).

18.6. Prove that $x = \cos x$ for some x in $(0, \pi/2)$.

18.7. Prove that $x2^x = 1$ for some x in $(0, 1)$.

18.12. Let $f(x) = \sin(1/x)$ for $x \neq 0$ and let $f(0) = 0$.

(a) Observe that f is discontinuous at 0 by Exercise 17.10(b).
(b) Show that f has the intermediate value property on $\mathbb{R}$.