Core Macroeconomics III

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Problem Set 1

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Problem 1: Two Period Consumption-Saving Problem with Exponential Utility

For this problem, I want you to re-do the consumption-savings problem under certainty presented in class assuming that that the utility function of the *exponential family*:

$$U(c) = -\frac{1}{\alpha}e^{-\alpha c}, \alpha > 0 \tag{1}$$

a) Draw a picture of this utility function over positive c, and verify that this utility function is increasing and concave. Note that there is nothing wrong with having utility be negative for all positive c – remember that the purpose of a utility function is ordinal, not cardinal.

b) Set up the maximization problem, derive the exact form of the Euler equation with this utility function, and derive a closed-form solution for optimal consumption in period 1. Assume capital markets are perfect.

Problem 2: T-period Consumption-Saving Decision Under Certainty with Perfect Capital Markets

The agent's preferences are given by:

$$\sum_{t=0}^{T} \beta^t U(c_t) \tag{2}$$

where $U(c)$ is of the CES/CRRA/Power Utility family:

$$U(c) = \frac{\sigma}{\sigma - 1} c^{\frac{\sigma-1}{\sigma}} \tag{3}$$

The agent's dynamic budget constraint (DBC) each period is given by

$$a_{t+1} = (1 + r)(a_t + y_t - c_t) \tag{4}$$

where r is the constant interest rate, a_t is the amount of wealth at the beginning of period t, and y_t is labor income in period t. To interpret (4), suppose that the agent begins period t with a positive wealth level a_t, and then gets additional resources in the form of new labor income y_t. If the agent consumes less than her current resources $(a_t + y_t)$ at time t, then she puts the unconsumed resources in savings and earns a return $(1 + r)$, so that a_{t+1} is positive. If the agent consumes more than her current resources, she must borrow and thus begins next period with negative resources $(a_{t+1} < 0)$, equal to (minus) the amount of debt the agent must repay.

Assume that the agent begins life with some initial inherited wealth a_0, in addition to any labor income y_0 she might receive in the first period. Notice that solvency requires that $a_{T+1} \geq 0$; the agent can be in debt during her life, but must pay off all of her debts by the end of period T. Assume that the agent exhausts all of her lifetime wealth, so that $a_{T+1} = 0$.

a) Show that the DBC (4), combined with the terminal condition $a_{T+1} = 0$, imply the following intertemporal budget constraint (IBC):

$$\sum_{t=0}^{T} \frac{c_t}{(1+r)^t} = a_0 + \sum_{t=0}^{T} \frac{y_t}{(1+r)^t} \equiv w_0 \tag{5}$$

(Hint: begin with the DBC describing a_1. Then substitute this expression for a_1 into the DBC for period 2, then substitute the resulting expression for a_2 into the DBC for period 3, and so on).

b) Set up the agent's maximization problem, derive the Euler Equation, and find a closed-form solution for c_0.