Core Macroeconomics III

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Problem Set 3

Spring 2006

Due Date: 04/27/05

Problem 1: The Life-Cycle Hypothesis and Durable Goods

So far in this term we have looked at models in which consumption goods yield utility only in the period in which they are bought. This assumption may be appropriate for nondurable goods and services, but it is clearly not appropriate for consumer durable goods such as autos and houses, since a durable good purchased in period t will typically yield flow utility at time t and for several periods thereafter.

Assume that a consumer's utility depends on the flow of services from a stock of durable goods. The consumer problem is

$$max \sum_{t=0}^{\infty} \beta^t u(k_t) \tag{1}$$

s.t.

$$k_t = c_t + (1-\delta)k_{t-1} \tag{2}$$

$$a_{t+1} = (1+r)(a_t + y_t - c_t) \tag{3}$$

$$a_0; k_0 \tag{4}$$

$$a_{T+1} = 0 (NPG) \tag{5}$$

where k_t is the consumer's stock of durable goods, c_t is purchases of durables, y_t is labor income, a_t is financial wealth, and $\delta \in (0,1)$ is the depreciation rate of durables. Assume that y_t is random as of $t-1$, that the riskless interest rate r is constant, and that there is no risky asset.

We can restate the consumer's problem in dynamic programming form, as follows (note that k_t is not a state variable at t, but k_{t-1} is):

$$V_t\left(a_t, k_{t-1}\right) = \max_{c_t} \left\{u(k_t) + \beta E_t\left[V_{t+1}(a_{t+1}, k_t)\right]\right\} \tag{6}$$

s.t. (2) and (3).

a) What additional assumptions would you have to make in order for the value function $V_t(a_t, k_{t-1})$ to be time -invariant?

b) Use the Bellman Equation (6) to establish the following Euler Equation for the stock of durable goods:

$$u'(k_t) = \beta(1+r)E_t u'(k_{t+1}) \tag{7}$$

Hints: the first order condition for c will involve $u'(k)$ and the expected derivatives of V with respect to both a and k. Evaluate the derivatives of V using the envelope theorem. Show that $V_k = (1-\delta)V_a$, and use this to eliminate the term involving dV/dk from the first order condition for c. Then show that $V_a = (1+r)\frac{u'(k)}{r+\delta}$.

c) Now assume that $u(k)$ is quadratic and that $\beta(1+r) = 1$. Show that the stock of durable follows a random walk:

$$k_{t+1} = k_t + \epsilon_{t+1} \tag{8}$$

where $E_t\epsilon_{t+1} = 0$. Show that durables purchases obey

$$c_{t+1} = c_t + \epsilon_{t+1} - (1-\delta)\epsilon_t \tag{9}$$

where ϵ's in (8) and (9) are identical.

d) Makiw (1982, on your reading list) finds that aggregate durables purchases (c_t) in the post war US follow a random walk; given c_t, variable such as c_{t-1}, $(c_t - c_{t-1})$, k_t, and $(k_t - k_{t-1})$ do not help predict c_{t+1}. Is this result consistent with (8) and (9)? (Note: for consumer durables, a reasonable value for δ is 0.3).

Problem 2: Consumption with Variable Labor Supply

Consider the problem of a household that has to choose both consumption and labor supply in a stochastic dynamic environment. The household problem is

$$V_0 = \max_{c_t, l_t} E_0 \sum_{t=0}^{T} \beta^t u(c_t, l_t) \tag{10}$$

s.t.

$$a_{t+1} = (1+r)(a_t + w_t l_t + I_t - c_t) \tag{11}$$

and a_0 is given, and $a_{T+1} = 0$, where l_t is the household's hours worked at t, w_t is the wage rate, I_t is transfer income, and within-period preferences are given by

$$u(c_t, l_t) = log(c_t) - \frac{B}{\gamma} l_t^{\gamma} \tag{12}$$

where $B > 0$ (so the household gets disutility from working) and $\gamma > 1$. The household regards l as a choice variable, while w and I are seen as exogenous. Assume that the household knows w_t and I_t at time t, but not at time $t-1$ (in other words, w and I are stochastic). For convenience, note that there is no risky assets and that the interest rate is constant; you may go further to assume that $\beta(1+r) = 1$.

a) Warm up question: show that $\gamma > 1$ insures that the marginal disutility of work is increasing in hours worked. Does this assumption makes sense?

b) Derive the following three conditions for optimal behavior by the household: (i) a 'static' first order condition relating today's labor supply to today's wage and today's consumption; (ii) an Euler Equation relating today's consumption to expectations involving tomorrow's consumption; and (iii) an Euler Equation relating today's labor supply to today's wage and expectations involving tomorrow's labor supply and wage.

c) Suppose we receive an unexpected increase in transfer income I at time t. Suppose this shock is expected to be permanent. Assuming the wage is unaffected, how will current labor supply be affected? What if the shock is expected to be transitory?

d) Now suppose that there is no uncertainty in the model. Suppose we observe that people's wages rise steadily over their lifetimes, but that hours worked are roughly constant over their life cycle. What does this tell us about the magnitude of γ?

Problem 3: Precautionary Saving with CARA Utility

To solve this problem you will need to know the following:

Definition: A random variable X is *lognormally distributed* if $log(X)$ is normally distributed.

Fact: If X is lognormal, then the mean of X satisfies

$$E(X) = e^{\mu + \frac{\sigma^2}{2}} \tag{13}$$

where (μ, σ) are the mean and standard deviation of $log(X)$.

Now the problem. Assume that the individual faces the following two-period problem, with zero discount and interest rates:

$$\max_{c_1, c_2} \{U(c_1) + E_1 U(c_2)\} \tag{14}$$

s.t.

$$c_2 = (w_1 - c_1) + w_2 \tag{15}$$

Assume that w_1 is known at time t, while w_2 is normally distributed with mean μ_w and variance σ_w^2. Assume that utility is CARA:

$$U(c) = -\frac{e^{-ac}}{a} \tag{16}$$

Derive a closed-form solution for first period consumption and first period precautionary saving. How does the amount of precautionary saving vary with the variance of w_2? How does precautionary saving and the marginal propensity to consume out of w_1 vary with w_1? How do these results compare to what Zeldes found for CRRA utility?

Problem 4: The LCH and Retirement

Suppose you are given the following data for a group of single men, aged 60 in year 1980 and 70 in year 1990: (1) wealth, income (broken down into labor income, pension income, and asset income) and nondurables spending in 1980 and 1990; and (2) the men's expectations, as of 1980, of annual pension income after retirement. Assume that there are no publicly provided pensions and that there is no other transfer income, but that some employers provide pensions to their workers after retirement; as of 1980, some men expect to receive a generous pension after retirement, while others expect a smaller pension or no pension at all. Assume that all men in this economy retire at age 65, so that everyone in your sample is working in 1980 and alive but retired as of 1990.

a) Explain how one could test the (quadratic utility) life cycle hypothesis under uncertainty using data on spending in 1980 and 1990, labor income in 1980 and pension income in 1990. If we rejected the LCH how could we distinguish myopia from liquidity constraints?

b) Suppose that people have different time-discount rates, and that people who are impatient tend to self-select into jobs with high earnings while working, but low pensions after retirement. Explain how this phenomenon could lead us to falsely reject the life cycle hypothesis in part (a). [HINT: this is an example of omitted variables bias– ideally, we would control for heterogeneous discount rates in our regression, but assume that we have no data that would allow us to do this].

c) Now suppose you also have data on spending for this group of men in 1970. Explain how we could use the change in spending between 1970 and 1980 to correct for the potential bias described in part (b).