Core Macreoeoconmics III

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Problem Set xx

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Due Date: xx/xx/xx

Important: Please attach all computer programs (codes) used to do this exercise.

Problem 1: Computational Exercise

Consider the following RBC model. The social planner chooses $\{C_t, K_{t+1}, N_t\}$ to maximize

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln (C_t) + \theta (1 - N_t) \right] \right\} \tag{1}$$

subject to

$$Y_t = (A_t N_t)^{\alpha} K_t^{1-\alpha} \tag{2}$$
$$Y_t = C_t + I_t + G_t \tag{3}$$
$$K_{t+1} = (1 - \delta) K_t + I_t \tag{4}$$
$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{5}$$
$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{6}$$

for $a_t = \ln (A_t)$, $g_t = \ln (G_t)$. Notation: C_t denotes consumption in period t, Y_t represents output in period t, N_t denotes labor during period t, K_t is the stock of capital in period t, and I_t represents investment in period t. a_t and g_t represent the processes for the stochastic shocks to productivity, A_t, and government expenditures, G_t. The intertemporal discount factor is $\beta > 0$, and the rate of depreciation of the capital stock is given by δ.

a) Show that the optimality conditions satisfy

$$C_t^{-1} = E_t \left\{ \beta R_{t+1} C_{t+1}^{-1} \right\} \tag{7}$$
$$\theta C_t = \alpha Y_t / N_t \tag{8}$$

where $R_t = (1 - \alpha) Y_t / K_t + (1 - \delta)$.

Interpret these conditions.

b) What are the log-linearized equations that describe the model?

c) Let $X_t = [c_t, y_t, n_t, k_t]'$, where lower-case letters represent log-linear deviations from steady state. Using matrix notation, express the model in the form

$$\hat{C} * X_t = \hat{A} * X_{t-1} + \hat{B} * E_t[X_{t+1}] + D_1 * w_t + D_2 * w_{t+1} \tag{9}$$

$$w_t = \Gamma * w_{t-1} + v_t \tag{10}$$

Clearly define $\hat{A}, \hat{B}, \hat{C}, D_1, D_2, \Gamma$.

d) Assume that $G_t = 0 \forall t$. Calibrate the model using $\delta = 0.025$, $\beta = 0.99$, and $\alpha = 2/3$. Let $\rho_a = 0.95$. Choose θ so that the hours worked are $1/3$ (note that this value won't affect dynamics here). Compute the numerical solution to this model. Use the solution to analyze the effect of shocks to ε_t^a on the economy. Plot the impulse response functions to consumption, investment, output, labor, interest rates, and wages in the model to a technology shock. Is labor productivity pro-cyclical or counter-cyclical in response to the shock? Explain the intuition behind your results.

e) Choose an appropriate value for $\sigma_a = E(\varepsilon_t^a)$. Compute a stochastic simulation of all variables in the model over 10,000 time periods. Use this stochastic simulation to compute the unconditional second moments of output, consumption, and investment. Also compute the correlations between the endogenous variables. Report these results. Is consumption more or less volatile than output?

f) Now assume that $A_t = 1 \forall t$, but allow G_t to be stochastic. Let the steady state $G/Y = 0.2$. Let $\rho_g = 0.95$. Redo exercise (d) and (e) for government spending shocks using an appropriate value for $\sigma_g = E(\varepsilon_t^g)$ in part (e). Based on these results, compare them with the previous ones.