411/511 Advanced Micro, Fall Term, Problem Set 1 **Peter Lambert**

WEEK 2: assigned exercises

to be handed in on the thursday of week 3

1. **Exercise N 3.8c / NS 3.10c** *(version from* **N** *reproduced overleaf)*

Here the utility function is of the Stone-Geary type $U(x,y) = (x-x_o)^{\alpha}(y-y_o)^{\beta}$ just as we had in class. The question wants you to obtain the MRS and decide whether it depends only on the ratio of the amounts of the two "goods", not on the actual quantities of the "goods". Try this first, using x and y as the two goods, and then try it again interpreting $X = (x-x_o)$ and $Y = (y-y_o)$ as the two "goods". (This utility will be homothetic in one case and not the other, all depending on what you interpret as the two "goods").

2. **Exercise N/NS 4.2** *(reproduced overleaf)*

Take advantage of the known fact about Cobb-Douglas utility functions, that this person will spend 2/3 of her budget on F and 1/3 on C. This should make (a) and (b) very easy. For (c), the question is really this: *how much extra income would this person have needed to achieve part (b) utility with part (a) prices?* This income increment is known as the *equivalent variation* for the given price change

3. **Exercise N 4.10 / NS 4.12** *(version from* **N** *reproduced overleaf)*

This is like the one I did in class for the utility function $U(x,y) = (x-x_o)^{\alpha}(y-y_o)^{\beta}$ where, by writing $X = (x-x_o)$ and $Y = (y-y_o)$ and adjusting the budget constraint, it came down to a standard Cobb-Douglas format. This one should go similarly ... over to you!

4. **An exercise *not* from the book:**

Let $U(x,y) = [\alpha x^{\rho} + \beta y^{\rho}]^{1/\rho}$ $\rho \leq 1, \rho \neq 0$ be the CES utility function we have been dealing with.

(a) Verify, using the result in the lecture notes for MRS_{xy}, that the elasticity of y/x with respect to MRS_{xy} along an IC for this utility function is $1/(1-\rho)$ (which we call σ);

(b) Use the fact that, when maximizing CES utility subject to the usual budget line, the optimal choices x^* and y^* satisfy

$$p_x x^* = \left[\frac{1}{1+\left(\frac{\beta}{\alpha}\right)^{\sigma}.\left(\frac{p_x}{p_y}\right)^{\sigma-1}}\right].I \quad \& \quad p_y y^* = \left[\frac{1}{1+\left(\frac{\alpha}{\beta}\right)^{\sigma}.\left(\frac{p_y}{p_x}\right)^{\sigma-1}}\right].I$$

to compute the indirect utility function $V(p_x, p_y, I)$. In order to avoid potentially horrendous algebra, answer this question for the special case in which $\boldsymbol{\alpha = \beta = 1}$ and $\boldsymbol{\rho = ½}$ only. You will still need to unscramble some algebra – but not much – and the answer to part (c) below might give you a clue as to what to aim for ...

(c) Confirm that for this utility function and these parameter values, the expenditure function takes the very simple form $E(p_x, p_y, \bar{u}) = \dfrac{p_x p_y \bar{u}}{p_x + p_y}$.

These questions are reproduced from **N** in case you haven't yet acquired either **N** or **NS** yet. *You need to obtain one of these books!*

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3.8

Example 3.3 shows that the *MRS* for the Cobb-Douglas function

$$U(x, y) = x^{\alpha} y^{\beta}$$

is given by

$$MRS = \frac{\alpha}{\beta}(y/x).$$

a. Does this result depend on whether $\alpha + \beta = 1$? Does this sum have any relevance to the theory of choice?

b. For commodity bundles for which $y = x$, how does the *MRS* depend on the values of α and β? Develop an intuitive explanation of why if $\alpha > \beta$, $MRS > 1$. Illustrate your argument with a graph.

c. Suppose an individual obtains utility only from amounts of x and y that exceed minimal subsistence levels given by x_0, y_0. In this case,

$$U(x, y) = (x - x_0)^{\alpha} (y - y_0)^{\beta}.$$

Is this function homothetic? (For a further discussion, see the Extensions to Chapter 4.)

4.2

a. A young connoisseur has $300 to spend to build a small wine cellar. She enjoys two vintages in particular: a 1997 French Bordeaux (w_F) at $20 per bottle and a less expensive 2002 California varietal wine (w_C) priced at $4. How much of each wine should she purchase if her utility is?

$$U(w_F, w_C) = w_F^{2/3} w_C^{1/3}$$

b. When she arrived at the wine store, our young oenologist discovered that the price of the French Bordeaux had fallen to $10 a bottle because of a decline in the value of the franc. If the price of the California wine remains stable at $4 per bottle, how much of each wine should our friend purchase to maximize utility under these altered conditions?

c. Explain why this wine-fancier is better off in part (b) than in part (a). How would you put a monetary value on this utility increase?

4.10

Suppose individuals require a certain level of food (x) to remain alive. Let this amount be given by x_0. Once x_0 is purchased, individuals obtain utility from food and other goods (y) of the form

$$U(x, y) = (x - x_0)^{\alpha} y^{\beta}$$

where $\alpha + \beta = 1$.

a. Show that if $I > p_x x_0$ the individual will maximize utility by spending $\alpha(I - p_x x_0) + p_x x_0$ on good x and $\beta(I - p_x x_0)$ on good y. Interpret this result.

b. How do the ratios $p_x x/I$ and $p_y y/I$ change as income increases in this problem? (See also Extension E4.2.)