Corrections to

Cohen–Macaulay rings

Revised Edition

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If you should find a mistake in the first or revised edition of our book, mathematical or typographical, please let us know by e-mail to

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However, we will list only those mistakes that have not been corrected in the revised edition. All data below refer to it.

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p. 105, proof of 3.2.13, l. 2 then $\operatorname{Supp}(Rx) \subset \operatorname{Supp} E = \{\mathfrak{m}\}$, see 3.2.5.

p. 110, l. 7–13 Replace these lines by the following:

> and so $\operatorname{Hom}_R(C, C')$ is cyclic by Nakayama's lemma. Let φ be a generator of this module. Then the natural epimorphism $R \to \operatorname{Hom}_R(C, C')$, $1 \mapsto \varphi$, induces the above isomorphism modulo $\boldsymbol{x}$. By 3.3.3, $\operatorname{Hom}_R(C, C')$ is a maximal Cohen–Macaulay module. Thus 3.3.2 implies that $R \to \operatorname{Hom}_R(C, C')$ is an isomorphism.

p. 112, l. 6–12 These lines contain an argument showing that multiplication by $a \neq 0$ cannot be the zero map on $E_R(k)$. The argument can be replaced by a reference to 3.2.12(e)(ii).

p. 129, 3.4.5(b) One has $H^i_{\mathfrak{m}}(M) = 0$ if $i < \operatorname{depth} M$.

p. 143, 3.6.18, l. 4 ${}^*H^i_{\mathfrak{m}}(M) \cong H_{\mathfrak{m}R_{\mathfrak{m}}}(M_{\mathfrak{m}})$

p. 214 l. 3 $\bar{R} = R/\boldsymbol{x}R$

p. 216 l. 19 only if $x < y$

p. 219, l. 9 $P_j(t) = 1 + t + \cdots + t^{r_j - 1}$

p. 228, *Shellings*, l. 3 $F_j \cap \bigcap_{i=1}^{j-1} F_i$

p. 228, paragraph below 5.2.14, l. 2 $P = \{x \in \mathbb{R}^d : \langle a_i, x\rangle \le 1,\ 1 \le i \le m\}$

p. 339, l. 1 *The reduction to the regular case.*

p. 360, proof of 9.3.2 We claim there is a minimal prime ideal $\mathfrak{p}$ such that height $\mathcal{O}(x) = \operatorname{height}((\mathcal{O}(x) + \mathfrak{p})/\mathfrak{p})$. In general, this is not true, as was pointed out to us by D. Eisenbud and B. Ulrich. (For unaccountable reasons we forgot to correct this mistake.)

Fortunately it is not difficult to solve the problem. First one passes to the completion $\hat{R}$ of R and replaces M by $M \otimes \hat{R}$. This causes no problems for height $\mathcal{O}(x)$ and big rank(M) since the extension $R \to \hat{R}$ is faithfully flat. Set $I = \mathcal{O}(x)$, and note that in view of what has to be proved it is enough to find a minimal prime ideal $\mathfrak{p}$ with $\operatorname{height}(I + \mathfrak{p})/\mathfrak{p} \ge \operatorname{height} I$. We simply choose a prime ideal $\mathfrak{p}$ with $\dim R = \dim R/\mathfrak{p}$. For all prime ideals $\mathfrak{q}$ of R with $\mathfrak{q} \supset I + \mathfrak{p}$ one then has

$$\operatorname{height} \mathfrak{q}/\mathfrak{p} = \dim R/\mathfrak{p} - \dim R/\mathfrak{q} \ge \dim R - \dim R/I \ge \operatorname{height} I,$$

as desired. The first equation uses that R is catenarian.

p. 380, 10.1.3(b), l. 4 only if char $k \ge 7$

p. 422, [56] The title of the paper is ‘On multigraded resolutions’.

p. 422, [57] The article has appeared: *J. Pure Appl. Algebra* **122** (1997), 185–208.

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p. 130 In lines 2–3 we require $\boldsymbol{x}$ to be a system of parameters. However, for the construction of the complexes $C^\bullet$ it is only necessary that $\boldsymbol{x}$ generates an $\mathfrak{m}$-primary ideal, and later on we use this fact several times.

p. 232, l. −4 as in Section 3.5

p. 305, 7.1.8 In (a) the correct hypothesis for $H'A$ is that it is generated as a B-module by the standard monomials that contain a factor from H'.

Part (c) of the exercise is wrong in the generality stated, but it holds for ASLs.

p. 354, 9.1.15 Add the hypothesis that N is of finite projective dimension.

p. 423, [74] The correct initial of Procesi is C.

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p. 268 In the last part of the proof of 6.2.5 one only needs to prove that $H^i(L^\bullet \otimes M) = 0$ for $i > 0$ if M is an injective R-module. Furthermore the case $\mathfrak{p} = \mathfrak{m}$ has been forgotten, but it is evident that $H^i(L^\bullet \otimes E(R/\mathfrak{m})) = 0$ for $i > 0$. If $\mathfrak{p} \neq \mathfrak{m}$, then $H^i(L^\bullet \otimes E(R/\mathfrak{p})) = 0$ for all $i \geq 0$ as shown.

p. 272 "If $z \in C$ ...": actually $\tilde{\mathcal{C}}(\mathcal{S})$ has not been defined for $\mathcal{S} = \emptyset$. So define it to be the zero complex.

The last index in the long exact sequence in (iii) should be $i - 1$.

pp. 308, 309 The correct Plücker relation is

$$\begin{aligned}&[146][235] + [124][356] - [134][256]\\ &\qquad + [126][345] - [136][245] - [123][456] = 0.\end{aligned}$$

This leads to the representation

$$[146][235] = -[123][456] - [125][346] + [135][246].$$

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p. 153 In Corollary 4.1.14(c) replace $\binom{n-d+i}{i}$ by $(n - d + i)!$.

p. 156 The k-algebras under consideration are finitely generated. (We have forgotten to require this condition explicitly.)

p. 158, l. 8 "...then $L(f) = \sum g_i L(f_i)$ ...": replace this by "$L(f) = gL(f_i)$ for some monomial g and some i"; replace $\sum g_i L(f_i)$ by $g f_i$ in the following.

p. 160, ll. 12/13 ...induction on d. For $d = 1$ ...that $d > 1$ ...

p. 161, l. −9 delete ", and $u \in S$ a monomial"

p. 162, l. 22 One can use 4.2.13 in order to derive the inequality $h(n) \leq \binom{n+m-1}{n}$.

p. 162, l. −5 Shorten the formula defining $a_{\langle d \rangle}$ by

$$a_{\langle d \rangle} = \binom{k(d)-1}{d} + \cdots + \binom{k(j)-1}{j},$$

where $j = \min\{i \colon k(i) \geq i\}$. (The version in the text is incorrect if $k(1) = 0$.) Furthermore add the definition $0_{\langle d \rangle} = 0$.

p. 166 In line 4 of the proof of 4.2.14 replace $\mathcal{M}_\backslash$ by $\mathcal{M}_n$. In the next line replace $I_n = 0$ by $J_n = 0$.

p. 168 In 4.2.17 one has to require that $\dim R > 0$.

p. 261, l. 3 $\operatorname{Ker} \varphi \cap \mathbb{Z}C$ is contained in the normalization of C. Since C is positive, the normalization is positive as well, and one concludes that φ is injective.

p. 384, l. −4 $a_1^q, \dots, a_j^q$ should also be multiplied by c.

p. 394 The proof of 10.3.3(a) has a gap. (This gap exists already in [215].)

From 10.1.19 one concludes (as in [215]) that $cr^q \in J_{i-1}^{[q]} : x_i^q \subset (J_{i-1}^{[q]})^*$ for q large, but this is not sufficient for $r \in J_{i-1}^*$.

As C. Huneke suggested, one should prove 10.3.3(a) by the same method as 10.1.9 (and the best solution would be to derive 10.1.9 from it). We write $R = A/I$ as in 10.1.9, and choose $z_1, \dots, z_g, y_1, \dots, y_d$ according to 10.1.10. Then $z_1, \dots, z_g, y_1^q, \dots, y_d^q$ is an A-sequence for all $q = p^e$, $e > 0$.

Again we set $J = (z_1, \dots, z_g)$, and use the notation $\mathfrak{p}_1, \dots, \mathfrak{p}_m, \mathfrak{p}_{m+1}, \dots, \mathfrak{p}_n$ as in the proof of 10.1.9. There exist $d \in (\mathfrak{p}_{m+1} \cap \dots \cap \mathfrak{p}_n)^s \setminus (\mathfrak{p}_1 \cup \dots \cup \mathfrak{p}_m)$, s sufficiently large, such that $dI^s \subset J$ (the letters r and c are already in use). Choose a power $q' \geq s$ of p.

For q large we have a relation

$$c'(r')^q y_i^q - (a_1 y_1^q + \dots + a_{i-1} y_{i-1}^q) \in I$$

where c' and r' are preimages of c and r in A. Take the q'-th power with $q' \geq s$ and multiply by d. Then one obtains an equation

$$d(c')^{q'}(r')^{qq'} y_i^{qq'} - (a_1' y_1^{qq'} + \dots + a_{i-1}' y_{i-1}^{qq'}) = b_1 z_1 + \dots + b_g z_g.$$

Using that $z_1, \dots, z_g, y_1^{qq'}, \dots, y_d^{qq'}$ is an A-sequence and taking residue classes modulo I yields

$$\tilde{d} c^{q'} r^{qq'} \in J_{i-1}^{[qq']}$$

for q large. Note that q' is *fixed*, and set $\tilde{c} = \tilde{d} c^{q'}$. Then $\tilde{c} \in R^\circ$, and $r \in (J_{i-1})^*$.

p. 395 In the proof of 10.3.5 one chooses t such that $(x_1^t, \dots, x_d^t) \subset (y_1, \dots, y_d)$ and writes $x_i^t = \sum_{j=1}^d a_{ij} y_j$

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p. 65, 2.1.29 One should additionally suppose that R is a graded $\mathbb{Z}$-algebra with $R_0 = \mathbb{Z}$. Then every homogeneous maximal ideal in $R \otimes \mathbb{Z}_\mathfrak{p}$ contains p. Note that a graded ring R is Cohen–Macaulay if and only if its localizations with respect to graded maximal ideals are Cohen–Macaulay (see 2.1.27).

Later on 2.1.29 is only applied in situations where R is even a positively graded $\mathbb{Z}$-algebra.

p. 141, 3.6.15(a) The a-invariant is $-\sum_{i=1}^{n} a_i$.

p. 269, 6.2.7(c) $\deg_{Y_1} f \neq 1$ and $\deg_{Y_2} f \neq 1$

p. 290, 6.4.15, l. 3 (a) $\Longrightarrow$ (b)

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p. 39, 1.5.21 I contains an R-regular sequence of length 2, but no such sequence of length 3,

p. 146, l. −15 $\operatorname{proj\,dim} R/I = 4$.

p. 146, l. −12 Remove the (repeated) sentence “The next case of interest is Gorenstein ideals of grade 4.”

p. 176, l. 6 One need not require k to be algebraically closed.

p. 293, 6.5.3.(c) dito

p. 280, proof of 6.4.2.(a) Delete the factor s_j from $s_j(u_{j1}a_1 + \ldots + u_{jn}a_n)$ in the second displayed formula.

p. 290, 6.4.16.(b)(ii) $S = U \cap \mathbb{N}^n$

p. 291, 6.4.17, l. 1 $R = k[Y_1, Y_2, Z_1, Z_2]$

p. 385, 10.1.12 the last item of the proposition should be labelled (e).

p. 403, l. −8 Replace $\mathcal{O}_X$ by $\mathcal{O}_W$ in the E_2^{pq} term.

p. 405, 10.3.28 $\max\{i : {}^*H_\mathfrak{m}^d(R)_i \neq 0\}$, $d = \dim R$

p. 424, 88 J. A. Eagon and V. Reiner. Resolutions of Stanley-Reisner rings and Alexander duality. J. Pure Appl. Algebra 130 (1998), 265–275 (1998)

p. 427, 149 N. Hara. A characterization of rational singularities in terms of injectivity of Frobenius maps. Am. J. Math. 120 (1998), 981–996.

p. 431, 234 T. Kawasaki. On Macaulayfication of Noetherian schemes. – Appendix A: d^+-sequences. Appendix B: An example. Trans. Am. Math. Soc. 352 (2000), No.6, 2517–2552; appendix A: 2541–2547; appendix B: 2548–2552.

p. 438, 391 N. Hara and K. Watanabe. F-regular and F-pure rings vs. log terminal and log canonical singularities. J. Algebr. Geom. 11 (2002), 363–392.

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p. 139, 3.6.9 The condition in (b) “provided $\mathfrak{m}$ is maximal” is superfluous. In fact, if $R/\mathfrak{m} \cong (R/\mathfrak{m})(-i)$, then there exists a homogeneous unit of degree i, and so every graded R-module M is isomorphic to $M(-i)$ in $\mathcal{M}_0(R)$.

p. 143, 3.6.18 In the last line of the remark the local cohomology module should be denoted by $H^i_{\mathfrak{m}R_\mathfrak{m}}(M_\mathfrak{m})$.

p. 157, l. −11 $I^\star = L(I)k[X_1, \dots, X_m]$

p. 171, l. 16 $H(R, j-2) - P_R(j-2) = -\dim_k {}^*H^1_\mathfrak{m}(R)_{j-2} < 0$